Incorporating Grade Uncertainty in Oil Sands Mine Planning and Waste Management Optimization Using Stochastic Mathematical Programming

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Abstract

The primary purpose of oil sands mine planning and waste management is to provide ore from the mine pit to the processing plant while containing the tailings in an efficient manner in-pit. Incorporating waste management in the mine plan is essential in maximizing the economic potential of the mineral resource and minimizing waste management cost. However, spatial variability such as grade uncertainty results in ore tonnage variations, which leads to variations in the quantity of waste to be managed. If grade uncertainty is not considered in oil sands mine planning, there may be excess waste than the waste management plan can handle or an over-design of a waste management system to handle less waste than planned. Both scenarios end up with lost opportunities. Conventional approaches to optimizing open pit mine production schedules are based on a single estimated orebody model which does not account for grade uncertainty. Grade uncertainties has profound impact on Net Present Value (NPV) of the mining project as it may induce large differences between the actual and expected production targets. Thus, the aim of this research is to develop an integrated oil sands mine planning optimization framework using Stochastic Mixed Integer Linear Programming (SMILP) to integrate the related domains of bitumen grade uncertainty and waste management. Sequential Gaussian Simulation (SGS) is employed to quantitatively model the spatial variability of bitumen grade in the oil sands deposit. Multiple simulated orebody models are used as inputs for the SMILP model to generate optimal results in the presence of grade uncertainty.

Keywords Oil sands mine planning, Production scheduling optimization, Stochastic mathematical programming, Sequential Gaussian Simulation, Waste management, Grade uncertainty

1 Introduction

Mine planning defines the source, destination, and sequence of extraction of ore and waste over the mine life. The result of mine planning is a production schedule that defines the tonnage of ore and waste and the input grade to the plant in each period of time. This production schedule has a significant influence on the economics of the mine due to time value of money. Improving production scheduling is essential as the mining industry considers more marginal resources. The natural complexity of mineral deposits makes mine planning more difficult. Moreover, the production schedule must follow physical and technical constraints, and meet the target capacity of the processing plant. Optimization algorithms are being applied in mine planning to maximize the overall profit of the project and minimize deviations from production targets. In traditional long-term mine planning, a geological block model is used as the main input to maximize the net present value (NPV) of the project. The geological block model is a quantitative definition of the available resource. Data from drill holes are used to construct the block model using geostatistical techniques.

Uncertainty in the generated block model data is inevitable with relatively widely spaced drill holes. The optimality of the open-pit production schedule will be affected by this uncertainty. Recent research initiatives have attempted to consider the effect of grade uncertainty on production schedules using mathematical programming. Mathematical programming formulations have the advantage of generating production schedules with a measure of the extent of optimality (Johnson, 1969; Gershon, 1983; Dagdelen, 1985; Dagdelen and Johnson, 1986; Akaike and Dagdelen, 1999; Caccetta and Hill, 2003). A major challenge in open-pit production scheduling with mathematical programming is the size of the optimization problem. The mathematical programming formulation of realistic long-term open-pit production schedules often exceeds the capacity of current hardware and optimization software (Badiozamani and Askari-Nasab, 2014). In this research, grade uncertainty is taken into account in an integrated long-term production scheduling and waste management. A case study based on oil sands deposit was evaluated and the results analyzed in this paper. A geological block model was first constructed for the oil sands deposit using geostatistical techniques such as ordinary kriging (OK) and sequential gaussian simulation (SGS). Thereafter, using the economic block model and production scheduling parameters, the project net present values (NPV) were determined and the scheduled results compared for the OK and SGS block models. The advantages of utilizing SGS models over the conventional OK models in grade estimation and mine planning are highlighted.

The rest of the paper is organized as follows; Section 2 describes the general process of oil sands mining and material handling strategies. Section 3 outlines the problem definition of this research Section 4 documents the research methodology that was followed and Section 5 discusses the mathematical programming formulation based on a Stochastic Mixed Integer Linear Programming (SMILP) framework for integrated oil sands mine planning and waste management. A case study is implemented and the results discussed in Section 6. The paper concludes in Section 7.

2 Oil Sands Mining

A typical oil sand mining system comprises of the removal of overburden material which is then preceded by mining the McMurray Formation containing the oil sands deposit. The first stage in any typical oil sands mining operation is to remove approximately 30 m of overburden material before the oil sand ore can be extracted. Each oil sands mining block is made up of ore, overburden (OB) and interburden (IB) dyke material, and waste. During extraction, the mined-out areas can be used as in-pit tailings storage areas as mining progresses in the specified direction and in-pit tailings dyke footprints are released. OB and IB dyke materials are used to construct facilities such as tailings dykes. These dykes are usually compacted by driving heavy trucks over them. Any other material that is not extracted as ore or dyke material, are sent to the waste dump. Ore material is hauled to the processing plant where it is crushed, slurried with warm process water and pumped to the extraction plant. (Mikula *et al.*, 1998; Chalaturnyk *et al.*, 2002; Soane *et al.*, 2010). The unsteady movement of oil sands ore during hydro transport breaks the ore into individual particles that are needed for separation in bitumen extraction. Process aids, dispersing agents and small air bubbles are then added before the ore reaches extraction stage. The oil sands ore is then mixed with water in a processing tower to create an oil sand slurry, which is then pumped to a central extraction plant where bitumen is extracted from the ore. After ore extraction, two main types of tailings are produced; (1) fine tailings and (2) coarse tailings. The fine tailings form the slurry which needs to be contained in the tailings facility.

According to the Alberta Energy Regulator (AER) (Alberta Energy Regulator, 2016), the minimum bitumen content of the oil sands material that will be classified as ore is 7 percent by weight of bitumen. A schematic view of the current oil sands mining material handling layout for long term mine planning could be seen in Figure 1.

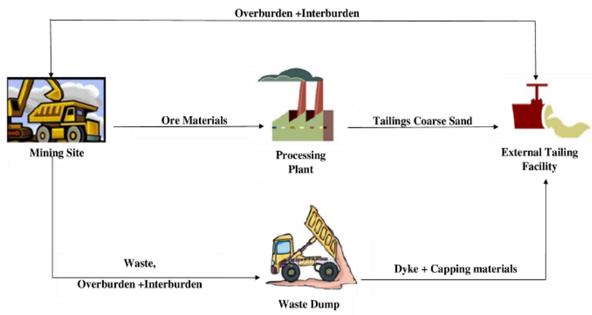


Figure 1 Oil sands mining material handling layout (modified after Badiozamani and Askari-Nasab, 2014)

3 Problem Definition

Conventional approaches to optimizing open pit mine production schedules are based on a single estimated orebody model which does not account for grade uncertainty. Grade uncertainties have a profound impact on the NPV and waste management strategy of the mining project as it may result in large differences between the actual and expected production targets, especially in the early years of mine life. Most methods developed to solve the mine production scheduling problem either ignore grade uncertainty or do not evaluate its impact on the waste management strategy.

The production scheduling of open-pit mines is an intricate, complex and difficult problem to address due to the large-scale optimization and the unavailability of a truly optimal net present value (NPV) solution. The complexity is increased by uncertainties due to the sparse variability of geological data. Godoy and Dimitrakopoulos (2004) classified the uncertainties involved in mine planning as (1) in-situ grade uncertainty and material type distribution; (2) technical mining specifications uncertainties such as extraction capacities and pit slope considerations; and (3) economic uncertainties including capital and operating costs. Osanloo et al. (2008) mentions several authors who consider the uncertainty of ore grade in long term production planning. They report that grade uncertainties result in metal accounting challenges since the metal content of the blocks are not known precisely at the time decisions are made but inferred from limited drilling information.

In addition to the impact of grade uncertainty on the long term mine plan, oil sands mining is increasingly becoming challenging as the public and lawmakers continue to put pressure on their waste management practices. Together with the limitations in lease areas, it has become necessary to look into effective and efficient waste disposal planning systems. These systems should be well integrated into the long term mine plan in an optimization framework that creates value and a sustainable operation. In oil sands operations, the pit phase mining occurs simultaneously with the construction of in-pit dykes in the mined-out areas of the pit and ex-pit dykes in designated areas outside the pit. These dykes are constructed to hold tailings that are produced during the processing of the oil sands ore. The materials used in constructing these dykes come from the oil sands mining operation. The dyke materials are made up of overburden (OB), interburden (IB) and tailings coarse sand (TCS). Dykes with different configurations are required during the construction. The material sent to the processing plant (ore) must have a specified minimum amount of bitumen and percentage fines, while material sent for dyke construction (dyke material) must meet the fines requirement for the dyke construction location. Any material that does not qualify as ore or dyke material is sent to the waste dump. A schematic representation of the problem definition for oil sands production mine planning and waste management could be seen in **Figure 2**

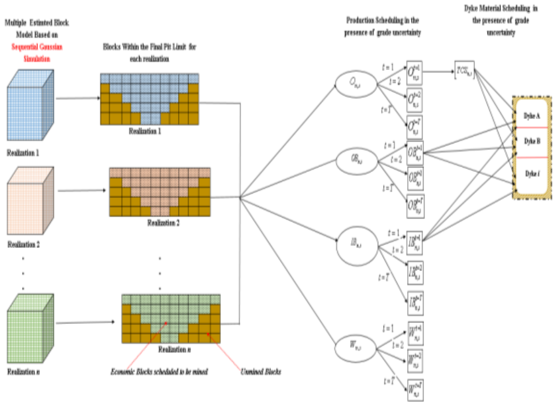


Figure 2 Schematic representation of the problem definition for oil sands production planning and waste management (modified after Ben-Awuah and Askari-Nasab, 2013)

In formulating an efficient model to incorporate grade uncertainties and waste management into oil sands mine planning, the research objectives can be characterized in three key areas:

a) Developing a risk-based integrated oil sands mine planning and waste management optimization framework using a Stochastic Mixed Integer Linear Programming (SMILP) model that will incorporate the related domains of bitumen grade uncertainty and robust oil sands waste management planning.

b) Determining the order and time of extraction of ore, dyke material, and waste to be removed from a predefined final pit limit over the mine life that maximizes the Net Present Value (NPV) of the operation.

c) Evaluating the risk profile associated with the mine plan and it impact on waste management.

4 Research Methodology

Grade uncertainty in a long-term production plan has been reported to affect the NPV of a mining project due to the differences between the actual and expected production targets especially in the early years of production (Osanloo *et al.*, 2008). Hence, to address the issue of grade uncertainty, instead of using a single estimated block model for production scheduling, twenty simulated realizations, that are representative of grade variability were used as input to the SMILP model. A conventional model which was based on Ordinary Kriging estimation was considered as the base case model. Both models including the E-type model were then compared to demonstrate the impact of grade uncertainty on the integrated mine production scheduling and waste management problem.

The workflow used in this research to generate an oil sands production schedule under grade uncertainty from the SMILP framework are as follows:

a) Create an oil sands geological and economical block models from the given drill hole data sets using GEOVIA GEMCOM software 6.7 (Gemcom Software International, 2012). The bitumen grades in this block model is estimated using Ordinary Kriging and serves as the base case model.
b) Implement geostatistical modelling using Sequential Gaussian Simulation (SGS) algorithm to map out bitumen ore grade uncertainty in the block model. In this step, Geostatistical Software Library known as GSLIB was used (Deutsch and Journel, 1998)
c) Determine the final pit limits using the base case block model, the E-type block model, and each realization block model using the 3D LG algorithm in GEOVIA Whittle software 4.7. (Gemcom Software International, 2012).
d) Export the blocks contained in the final pit limits for the base case model into GEOVIA GEMS software and using the pit design parameters, design the final pit outlines using the final pit limits as guide.
e) Select all the blocks that are within the designed final pit limits for all the realizations, E-type and Ordinary Kriging block models. Save the selected blocks within the final pit limits in ASCI file format.
f) For each case study, define the input scheduling parameters in MATLAB to formulate the problem.
g) Implement the developed mathematical programming formulations in MATLAB. TOMLAB/CPLEX (ILOG, 2017) is used as the solver for the defined optimization problem
h) Perform production scheduling optimization runs and comparative analysis based on the generated results from the OK, E-type and SGS block models.

5 Stochastic Mixed Integer Linear Programming (SMILP) Model Formulation

The SMILP optimization framework for generating long-term production schedule in the presence of grade uncertainty was modeled using multiple realizations from Sequential Gaussian Simulation (SGS). A parameter known as the Economic Block Value (EBV) is calculated for each block in each realization. The EBV of a block is the revenue generated by selling the final product less all the costs involved in extracting and processing the block. The mining cost of a block is a function of the distance between its location and its final destination. Since the long-term production plan is a multi-period optimization problem and blocks are extracted in different periods, a discount rate is applied to calculate the present value of the EBV, revenue and the costs. Therefore, the Discounted Economic Block Value (DEBV) for the stochastic model is calculated using Equation (1).

$$d_{n,s}^{u,t} = v_{n,s}^{u,t} - q_n^{a,t} - p_{n,s}^{u,t} - m_{n,s}^{u,t} - h_{n,s}^{u,t} \qquad (1)$$

The parameters stated in Equation (2) to Equation (6) are used to calculate the Discounted Economic Block Value. These parameters are defined by;

$$v_{n,s}^{u,t} = \sum_{e=1}^{E} o_{n,s} \times g_{n,s}^{e} \times r^{u,e} \times (P^{e,t} - cs^{e,t}) - \sum_{e=1}^{E} o_{n,s} \times cp^{u,e,t} \qquad (2)$$

$$q_n^{a,t} = (o_n + d_n + i_n + w_n) \times cm^{a,t} \qquad (3)$$

$$p_{n,s}^{u,t} = d_{n,s} \times ck^{u,t} \qquad (4)$$

$$m_{n,s}^{u,t} = i_{n,s} \times cb^{u,t} \qquad (5)$$

$$h_{n,s}^{u,t} = j_{n,s} \times ct^{u,t} \qquad (6)$$

The discounted revenue shown in Equation (2) is the present value of the ore minus the cost of processing the ore. The discounted cost of mining all the block material as waste is represented by Equation (3).

The extra discounted costs of mining materials such as overburden, interburden, and tailings coarse sands for the purpose of dyke construction are represented by Equation (4) to Equation (6) respectively. The notations used in the formulation of the SMILP model optimization problem have been classified as indices, sets, superscripts, subscripts and decision variables follows:

Indices

$a \in \{1,....,A\}$ index for possible mining locations (pits)

$e \in \{1,....,E\}$ index for the element of interest in each block

$j \in \{1,....,J\}$ index for phases (pushbacks)

$n \in \{1,....,N\}$ index for blocks

$s \in \{1,....,S\}$ index for SGS realizations

$t \in \{1,....,T\}$ index for scheduling periods

$u \in \{1,....,U\}$ index for possible destinations for materials

Sets

$A = \{1,....,A\}$ set of all index for possible mining locations (pits) in the model

$J = \{1,....,J\}$ set of all the phases in the model

$N = \{1,....,N\}$ set of all the blocks in the model

$S = \{1,....,S\}$ set of all equally probable orebody realizations

$U = \{1,....,U\}$ set of all the possible destinations in the model

$D_n(J)$ for each block, there is a set $D_n(J)$ which includes all the blocks that must be extracted prior to mining block *n* to ensure that block *n* is exposed for mining with safe pit slopes, where *J* is the total number of blocks in the set $D_n(J)$

$C_n(L)$ for each block, there is a set $C_n(L) \subset D_n(J)$ defining the immediate predecessor blocks that must be extracted prior to extraction of the block *n*, where *L* is the total number of blocks in the set $C_n(L)$

Decision variables

$a_n^t \in \{0,1\}$ binary integer variable controlling the precedence of extraction of blocks a_n^t is equal to one if extraction of block n has started by or in period *t*, otherwise, it is zero

$x_n^{u,t} \in [0,1]$ a continuous variable representing the ore portion of block *n*, that is to be to be extracted and processed at destination *u* in period *t*

$y_n^{a,t} \in [0,1]$ a continuous variable representing the portion of block *n* to be mined in period *t* from location *a*, which includes both ore, overburden, interburden and waste from the associated blocks

$c_n^{u,t} \in [0,1]$ a continuous variable representing the interburden dyke material portion of block *n* that is to be extracted and used for dyke construction at destination *u* in period *t*

$gdev_{s,-}^t$ a continuous variable representing the shortage of the grade upper bound in period *t* in realization *s*

$gdev_{s,+}^t$ a continuous variable representing the surplus of the grade lower bound in period *t* in realization *s*

$odev_{s,-}^t$ a continuous variable representing the shortage of the ore tonnage upper bound in period *t* in realization *s*

$odev_{s,+}^t$ a continuous variable representing the surplus of the ore tonnage lower bound in period *t*

$s_n^{u,t} \in [0,1]$ a continuous variable representing the tailing coarse sand dyke material portion of block *n* that is to be extracted and used for dyke construction at destination *u* in period t

$l_n^{u,t} \in [0,1]$ a continuous variable representing the overburden dyke material portion of block n that is to be extracted and used for dyke construction at destination u in period t

Parameters

$cb^{u,t}$ the cost in present value terms per tonne of interburden dyke material for dyke construction at destination u

$ck^{u,t}$ the cost in present value terms per tonne of overburden dyke material for dyke construction at destination u.

$ct^{u,t}$ the cost in present value terms per tonne of tailings coarse sand dyke material for dyke construction at destination u

$cm^{a,t}$ the cost in present value terms of mining a tonne of waste in period t from location a.

$cp^{u,e,t}$ the extra cost in present value terms per tonne of ore for mining and processing at destination u

$cs^{e,t}$ the selling cost of element e in present value terms per unit of product.

$d_n^{u,t}$ the discounted economic block value obtained by extracting block n and sending it to destination u in period t

$d_{n,s}$ overburden dyke material tonnage in block n

$dgeo$ geological discount rate

f_k^e the average percent of fines in ore portion of block n

$\underline{f}^{u,t,e}$ the lower bound on the required average fines percent of ore in period t at processing destination u

$\overline{f}^{u,t,e}$ the upper bound on the required average fines percent of ore in period t at processing destination u

f_n^d the average percent of fines in interburden dyke material portion of block n

$\underline{f}^{u,t,d}$ the lower bound on the required average fines percent of interburden dyke material in period t at dyke construction destination u

$\overline{f}^{u,t,d}$ the upper bound on the required average fines percent of interburden dyke material in period t at dyke construction destination u

$g_{n,s}^e$ the average grade of element e in ore portion of block n in realization s

$\underline{g}^{u,t,e}$ the lower bound on the required average head grade of element e in period t at processing destination u

$\overline{g}^{u,t,e}$ the upper bound on the required average head grade of element e in period t at processing destination u

$h_{n,s}^{u,t}$ the discounted cost of mining tailings course sand dyke material in block n of realization s and in period t for construction at destination u

$i_{n,s}$ the interburden dyke material tonnage in block n of realization s

$j_{n,s}$ the tailings coarse sand dyke material tonnage in block n of realization s

$m_{n,s}^{u,t}$ the extra discounted cost of mining all the material in block n of realization s and in period t as interburden dyke material for construction at destination u

$o_{n,s}$ the ore tonnage in block n of realization s

$p_{n,s}^{u,t}$ the extra discounted cost of mining all the material in block n of realization s and in period t as overburden dyke material for construction at destination u

$P^{e,t}$ the price of element e in present value terms per unit of product

pc_{g-}^{t} penalty cost for lower grade target deviation in period t

pc_{g+}^{t} penalty cost for upper grade target deviation in period t

pc_{o-}^{t} penalty cost for lower ore tonnage target deviation in period t

pc_{o+}^{t} penalty cost for upper ore tonnage target deviation in period t

$q_{n}^{a,t}$ the discounted cost of mining all the materials in block n in period t as waste from location a

r interest rate

$r^{u,e}$ the proportion of element e recovered if it is processed at destination u

$\underline{T_m}^{a,t}$ lower bound on mining capacity in period t at location a (tonnes)

$\overline{T_m}^{a,t}$ upper bound on mining capacity in period t at location a (tonnes)

$\underline{T_{pr}}^{u,t}$ lower bound on processing capacity in period t at destination u (tonnes)

$\overline{T_{pr}}^{u,t}$ upper bound on processing capacity in period t at destination u (tonnes)

$\underline{T_{od}}^{u,t}$ lower bound on overburden dyke material in period t at destination u (tonnes)

$\overline{T_{od}}^{u,t}$ upper bound on overburden dyke material in period t at destination u (tonnes)

$\underline{T_{id}}^{u,t}$ lower bound on interburden dyke material in period t at destination u (tonnes)

$\overline{T_{id}}^{u,t}$ upper bound on interburden dyke material in period t at destination u (tonnes)

$\underline{T_{td}}^{u,t}$ lower bound on tailing coarse sand dyke material in period t at destination u (tonnes)

$\overline{T_{td}}^{u,t}$ upper bound on tailing coarse sand dyke material in period t at destination u (tonnes)

sf_{n}^{e} the average ratio of sand-to-fines in ore portion of block n

$\underline{sf}^{u,t,e}$ the lower bound on the required average sand-to-fines ratio for ore in period t at processing destination u

$\overline{sf}^{u,t,e}$ the upper bound on the required average sand-to-fines ratio for ore in period t at processing destination u

$w_{n,s}$ the waste tonnage in block n

$v_{n,s}^{u,t}$ discounted revenue obtained by selling the final product within block n of realization s in period t if it is sent to destination u, minus extra discounted cost of mining all the material in block n as ore from location a and processing it destination u

5.1. SMILP Model Objective Function

The objective function for the SMILP model for integrated long term production planning and waste management is formulated in three main parts: 1) maximizing the net present value of the mining operation (Equation (7)), 2) minimizing the dyke construction cost for the waste management plan

(Equation(8)) and 3) minimizing the cost associated with deviating from the operating targets, including ore grade and ore tonnage deviations (Equations (9) and (10)). These control variability of grade targets and ore tonnage targets.

$$Max\frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{a=1}^{A}\sum_{u=1}^{U}\sum_{n=1}^{N}\left(\frac{v_{n,s}^{u,t}\times x_{n}^{u,t}-q_{n}\times y_{n}^{a,t}}{(1+r)^{t}}\right) \quad (7)$$

$$Min\frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{a=1}^{A}\sum_{u=1}^{U}\sum_{n=1}^{N}\left(\frac{p_{n,s}^{u,t}\times l_{n}^{u,t}+m_{n,s}^{u,t}\times c_{n}^{u,t}+h_{n,s}^{u,t}\times s_{n}^{u,t}}{(1+r)^{t}}\right) \quad (8)$$

$$Min\frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\left(\frac{pc_{o+}^{t}\times odev_{s,+}^{t}+pc_{o-}^{t}\times odev_{s,-}^{t}}{(1+dgeo)^{t}}\right) \quad (9)$$

$$Min\frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\left(\frac{pc_{g+}^{t}\times gdev_{s,+}^{t}+pc_{g-}^{t}\times gdev_{s,-}^{t}}{(1+dgeo)^{t}}\right) \quad (10)$$

In the objective function, the SMILP model consists of continuous and binary decision variables. Some continuous decision variables can take any value between 0 and 1 while the binary variable value can take either 0 or 1. The binary variable is used to control precedence of block extraction while the continuous variables such as $x_n^{u,t}$, $y_n^{a,t}$, $l_n^{u,t}$, $c_n^{u,t}$, $s_n^{u,t}$ control the portion of blocks that is to be extracted. Other continuous decision variables like *gdev* and *odev* provide range of acceptable deviation from ore grade and ore tonnages targets that will minimize the financial risks of not meeting the production targets

In Equation (7) there are two decision variables for each block n. These decision variables are $x_n^{u,t}$ and $y_n^{a,t}$. The first decision variable $x_n^{u,t}$ represents the portion of block n that is to be processed (if it is ore) in period t while the second decision variable $y_n^{a,t}$ represents the portion of block n that is to be extracted in period t. By using two different decision variables for extraction and processing of each block, the optimizer decides whether a block should be processed or sent to the waste dump. Therefore, cut-off grade is implemented implicitly in the optimization process. By using two decision variables, it is possible to generate a schedule that may send low quality ore blocks located on upper benches to the waste dump in order to gain access to high-quality ore blocks on the lower levels. This generates more revenue in early periods of the mine life increasing the total profit of the project.

In Equation (8), the aim is to minimize the cost of dyke materials extraction for dyke construction in line with waste management practices in oil sands mining. Continuous decision variables $l_n^{u,t}$, $c_n^{u,t}$, $s_n^{u,t}$ are used to control the overburden, interburden and tailings coarse sand dyke material portions of a block extracted for dyke construction. To integrate grade uncertainty modelled with SGS realizations, continuous deviation variables $gdev_{s,-}^{t}$, $gdev_{s,+}^{t}$, $odev_{s,-}^{t}$, and $odev_{s,+}^{t}$ are introduced in Equations (9) and (10) with their respective penalties pc_{g+}^{t}, pc_{g-}^{t}, pc_{o+}^{t} and pc_{o-}^{t} for managing deviations from ore grade and ore tonnage production targets. Also, a geological discount rate (*dgeo*) is applied to the cost of deviation to defer the risk of not meeting production targets to later periods.

5.2. SMILP Model Constraints for Production Scheduling

The related constraints used in controlling the mining and processing capacities are stated by Equations (11) to (14). These constraints are defined in the form of maximum and minimum limits and are controlled by the decision variables $y_n^{a,t}$ and $x_n^{u,t}$. Equation (11) and Equation (12) define the maximum and minimum mining capacity constraints while Equation (13) and Equation (14) define the processing capacity constraints. Ore tonnage uncertainty modelled with SGS realizations are taken into consideration in Equation (15) and (16) introduce the continous decision variables $gdev_{s,-}^{t}$ and $gdev_{s,+}^{t}$ which are used as buffers to allow grade deviations from the target head grade. Penalizing these deviations in the objective function (Equations (16) and (17)) ensures that both the proportion of ore processed at the plant and input grade fed to the mill are as close as possible to the required targets. The ore quality blending constraints used to control ore bitumen grades and ore fines content in the extracted materials for all realizations are formulated in Equations (17) to Equations (18).

$$\sum_{n=1}^{N}(o_n+d_n+i_n+w_n)\times y_n^{a,t}\le \overline{T}_m^{a,t} \quad (11)$$

$$\sum_{n=1}^{N}(o_n + d_n + i_n + w_n) \times y_n^{a,t} \ge \underline{T_m}^{a,t} \quad (12)$$

$$\sum_{n=1}^{N}\left(o_{n,s} \times x_n^{u,t}\right) - odev_{s,+}^{t} \le \overline{T_{pr}}^{u,t} \quad (13)$$

$$\sum_{n=1}^{N}\left(o_{n,s} \times x_n^{u,t}\right) + odev_{s,-}^{t} \ge \underline{T_{pr}}^{u,t} \quad (14)$$

$$\left(\sum_{n=1}^{N} g_{n,s}^{e}\left(o_{n,s} \times x_n^{u,t}\right) - \sum_{n=1}^{N} \overline{g}^{u,t,e}\left(o_{n,s} \times x_n^{u,t}\right) - gdev_{s,+}^{t}\right) \le 0 \quad (15)$$

$$\left(\sum_{n=1}^{N} g_{n,s}^{e}\left(o_{n,s} \times x_n^{u,t}\right) - \sum_{n=1}^{N} \underline{g}^{u,t,e}\left(o_{n,s} \times x_n^{u,t}\right) + gdev_{s,-}^{t}\right) \ge 0 \quad (16)$$

$$\frac{1}{S}\sum_{s=1}^{S}\left(\sum_{n=1}^{N} f_{n,s}^{e}(o_{n,s} \times x_n^{u,t}) - \sum_{n=1}^{N} \overline{f}^{u,t,e}(o_{n,s} \times x_n^{u,t})\right) \le 0 \quad (17)$$

$$\frac{1}{S}\sum_{s=1}^{S}\left(\sum_{n=1}^{N} f_{n,s}^{e}(o_{n,s} \times x_n^{u,t}) - \sum_{n=1}^{N} \underline{f}^{u,t,e}(o_{n,s} \times x_n^{u,t})\right) \ge 0 \quad (18)$$

5.3. SMILP Model Dyke Material Constraints for Risk-based Waste Management Planning

The presence of grade uncertainty in mineral deposits contribute to unsustainable waste management planning which may result in major financial liabilities, environmental challenges and mine closures by regulatory agencies. If grade uncertainty is not considered in oil sands mine planning, there may be excess waste than the waste management plan can handle or an over-design of a waste management system to handle less waste than planned. This results in lost opportunities in terms of revenue and waste management cost. In the proposed SMILP model, grade variability in the material extracted should be taken into consideration during production scheduling so as to avoid under- or over-representation of the proportions of materials classified as either ore or waste. By doing so, the classified waste materials can be represented appropriately in a robust waste management plan to minimize environmental impacts and costs.

The constraints used in controlling the OB, IB, and TCS dyke materials requirements and IB dyke material fines content targets are modelled with Equations (19) to (26) . From Equation (8) the constraints for dyke material requirements are controlled by the decision variables $l_n^{u,t}$, $c_n^{u,t}$, $s_n^{u,t}$. Equations (25) and (26) are IB dyke material fines blending constraints which controls the fines content in the extracted material for dyke construction destinations. These constraints control the IB dyke material fines to ensure stability of the tailings cell dyke walls during construction as per the dyke design specifications.

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{n=1}^{N} d_{n,s} \times l_n^{u,t} \le \overline{T_{od}}^{u,t} \quad (19)$$

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{n=1}^{N} d_{n,s} \times l_n^{u,t} \ge \underline{T_{od}}^{u,t} \quad (20)$$

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{n=1}^{N} i_{n,s} \times c_n^{u,t} \le \overline{T_{id}}^{u,t} \quad (21)$$

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{n=1}^{N} i_{n,s} \times c_n^{u,t} \ge \underline{T_{id}}^{u,t} \quad (22)$$

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{n=1}^{N} j_{n,s} \times s_n^{u,t} \le \overline{T_{td}}^{u,t} \quad (23)$$

$$\frac{1}{S}\sum_{s=1}^{S}\sum_{n=1}^{N} j_{n,s} \times s_n^{u,t} \ge \underline{T_{td}}^{u,t} \quad (24)$$

$$\frac{1}{S}\sum_{s=1}^{S}\left(\sum_{n=1}^{N} f_{n,s}^{d}(i_{n,s} \times c_n^{u,t}) - \sum_{n=1}^{N} \overline{f}^{u,t,d}(i_{n,s} \times c_n^{u,t})\right) \le 0 \quad (25)$$

$$\frac{1}{S}\sum_{s=1}^{S}\left(\sum_{n=1}^{N} f_{n,s}^{d}(i_{n,s} \times c_n^{u,t}) - \sum_{n=1}^{N} \underline{f}^{u,t,d}(i_{n,s} \times c_n^{u,t})\right) \ge 0 \quad (26)$$

$$\frac{1}{S}\sum_{s=1}^{S}\left(\sum_{n=1}^{N} sf_{n,s}^{e}\left(o_{n,s} \times x_n^{u,t}\right) - \sum_{n=1}^{N} \overline{sf}^{u,t,e}\left(o_{n,s} \times x_n^{u,t}\right)\right) \le 0 \quad (27)$$

$$\frac{1}{S}\sum_{s=1}^{S}\left(\sum_{n=1}^{N} sf_{n,s}^{e}\left(o_{n,s} \times x_n^{u,t}\right) - \sum_{n=1}^{N} \underline{sf}^{u,t,e}\left(o_{n,s} \times x_n^{u,t}\right)\right) \ge 0 \quad (28)$$

5.4. SMILP Model General Constraints

The general constraints that is widely used in mine optimization problems relates to controlling the mining precedence and the logics of the variables during optimization. These constraints have been formulated and documented in Ben-Awuah and Askari-Nasab (2013) as follows:

a) Vertical mining precedence: These constraints ensure that before the extraction of a specific mining block, all the immediate predecessor blocks on top must already have been extracted so that the mining block is accessible.
b) Horizontal mining precedence: These constraints ensure that all the immediate predecessor mining blocks in a specified horizontal mining direction are extracted prior to the extraction of the mining block.

c) Variables logics control: These constraints control the logics of the mining, processing, and dyke material variables with respect to their limits and definitions.

6 Case Study: Results and Discussions

The data set used for this case study is an oil sands deposit which consists of 104 exploration drill holes. **Figure 3** shows a 3D location map of the drill holes with bitumen grades.

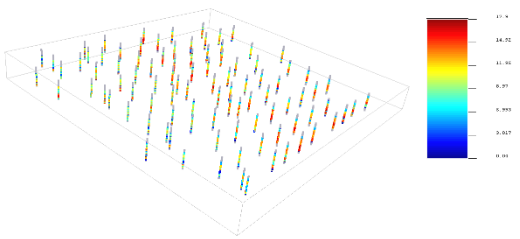


Figure 3 3D location map of the drill holes with bitumen grade distribution

6.1 Statistical Analysis of Oil Sands Data

In order to start geostatistical modelling, it is necessary to perform preliminary statistical analysis including compositing, recognizing outliers, identifying trends, and data transformation. The drill holes contain assay data for bitumen, fines, organic rich solids, and water. Declustering was applied to the data set to reduce the effect of clustered samples on global statistics. By applying the concept of declustering, the histogram and summary statistics are adjusted to be representative of the entire volume of interest. This is necessary because data collection practices in general focuses on portions of the study area that are most important. For this research, the element of interest is bitumen grades since its variability in estimation creates uncertainty which potentially impacts the overall net present value of the mining project. (Osanloo *et al*., 2008), (Ramazan and Dimitrakopoulos, 2018).

6.2 Spatial Correlation Analysis using Variogram

The measurement of spatial continuity was employed so as to understand the correlation between the observations of the univariate sample at different locations. This analysis is useful to detect the presence of general trends in the data. Geostatistical techniques as explained by Issaak and Srivastava (1989) were used to analyze spatial variability and distribution of sample data to estimate parameters at unsampled locations. According to Issaak and Srivastava (1989), applying geostatistics techniques has three steps: (1) assumption of stationarity, (2) spatial modelling of sample data, and (3) estimation of variable value at unsampled location. The analysis of spatial correlation can be undertaken using Geostatistical Software Library software also known as GSLIB (Deutsch and Journel, 1998).

The original data set containing bitumen grades were transformed to a gaussian space using standard normal score transformation applied in geostatistical analyses (Deutsch 2002). Transformation of data to normal score distribution satisfies the assumption of stationarity of data. The transformed normal score data is also useful as input data in the stochastic gaussian simulation technique. Figure 4 shows the histogram and the normal score transform for bitumen grades.

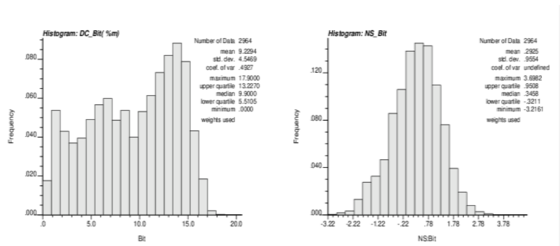


Figure 4 Histogram and transformed normal score for bitumen grades

Variogram analysis, which allows for examination of whether the data is correlated with distance, was done for ore bitumen grades. Omnidirectional variogram for bitumen grades was first computed to identify the sill while vertical variograms were used to identify the nugget effect. Primary variogram maps were calculated to determine the orientation of the major axis in the presence of anisotropy. Directional experimental variograms were calculated and theoretical variogram models were fitted to the experimental variograms. The parameters used to model the experimental variogram can be seen in Table 1.

Table 1 Parameters used for Variogram Modelling of Bitumen Grades

Direction	Azimuth	Variogram Model	Sill Contribution	h_{min} (m)	h_{max} (m)	Nugget
Vertical	N0.0S	Exponential	0.43	180	220	0.57
Minor	N22.5E	Exponential	0.58	320	800	0.42
Major	N67.5E	Exponential	0.18 0.01	610 1200	800 1600	0.81 0.19

An exponential variogram model was fitted to the experimental variograms in the horizontal major direction and the horizontal minor direction. Figure (5) shows the experimental and fitted variogram models in the major and minor directions, and in the vertical direction. The general equation for exponential variogram model is shown in Equation (29).

$$\gamma(h) = C \times \left[1 - e^{-\left(\frac{3h}{a}\right)^2}\right] \quad (29)$$

Where C is the structure variance and a is the effective range.

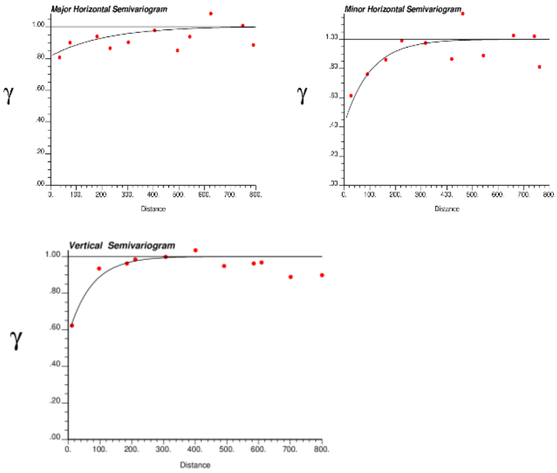


Figure 5 Experimental directional variogram (dots) and fitted variogram models (solid lines) for bitumen grades of ore blocks (distance in meters)

6.3 Estimation and Simulation

Using the concept of Block Kriging (Gringarten and Deutsch, 2001), Ordinary Kriging grade and variance were estimated for each mining block for bitumen and fines grades. Figure 6 shows a 2D plot of the OK mean and variance estimated for bitumen grades in the case study. Due to the smoothing effect of Kriging, the standard deviation decreases as compared to the sample data. The global average ore bitumen grade estimate from OK is 9.22 (%m).

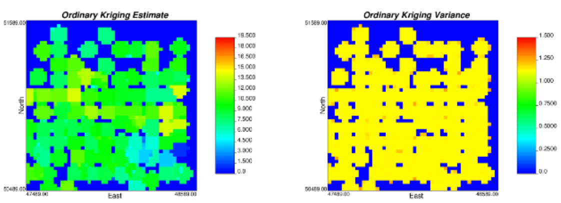


Figure 6 2D plot of OK mean (left) and OK variance (right) estimated for bitumen grades

Subsequently, the results of 20 SGS realizations for bitumen grades were obtained after running the stochastic sequential gaussian simulation algorithm. The results are shown in Figure 7.

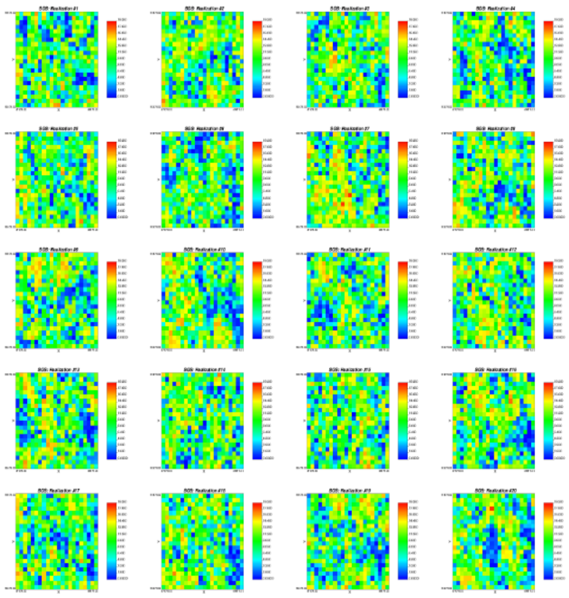
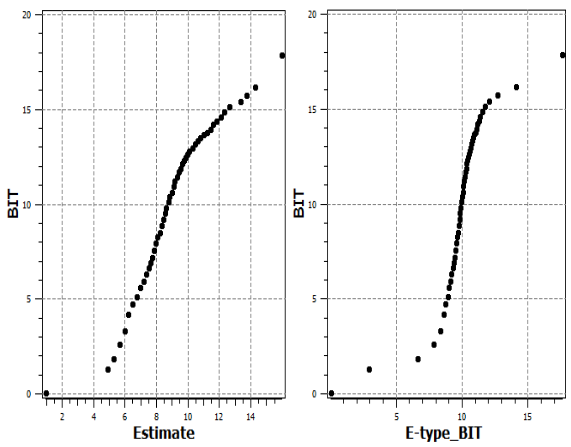


Figure 7 Simulated results obtained from SGS

In order to compare and validate the estimated kriging results and SGS results, a QQ-plot was created by plotting two sets of quantiles against each other. In this case, the estimated kriged bitumen grades and simulated bitumen grades were compared to the true bitumen grade values from the samples. The Q–Q plot analysis produced acceptable linear trends (located on the 45-degree line) between sample data and realizations. This shows good reproduction of sample data statistics from the simulation results. On the other hand, it can be seen that the Q–Q plot analysis for OK estimates deviates from the 45-degree trend line.



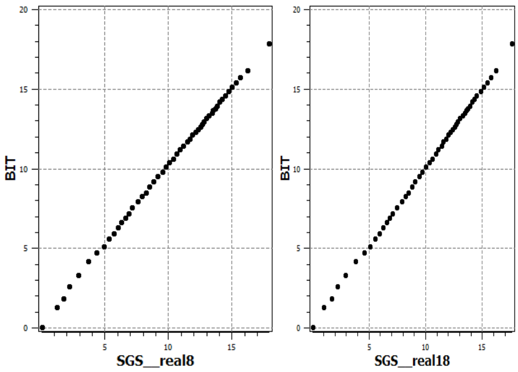


Figure 8 Plots of estimated and simulated ore bitumen grades against input data (OK, E-type, SGS realizations 8 and 18)

6.4 Implementation of the SMILP Framework

This section provides detailed documentation of the case study experimental design and implementation with the SMILP model framework for an oil sands deposit. In this case study, there were three scenarios that were implemented. Scenario 1 is based on the OK block model and is used as the base case study. Scenario 2 is based on the E-type block model and Scenario 3 is based on the SGS realizations block models. The SMILP framework is based on the application of all generated SGS realizations (Scenario 3) so as to consider grade uncertainty in the production schedule. The SMILP framework is also adjusted and applied in Scenarios 1 and 2 with the assumption that they each have one realization. Figure 9 shows the implementation scenarios that were investigated for this case study. A summarized information on the oil sands deposit which includes ore and waste materials contained in the ultimate pit design is presented in Table 2. The economic parameters that were used in the case study for all scenarios is shown in Table 3.The minimum and maximum limits of material quantity and quality requirements for ore bitumen grades, ore fines content, IB dyke material fines content, and sand-fines blends for all scenarios can be seen in Table 4 for all scenarios.

The risk parameters considered to minimize deviations of ore bitumen grades and ore tonnages from the production targets during mine production scheduling can be seen in Table 5.**Error! Reference source not found.** For Scenarios 1 and 2, there was no consideration of uncertainty. The OK model in Scenario 1 is said to produce the best linear unbiased estimate (Isaaks and Srivastava, 1989; Sinclair and Blackwell, 2002), while the E-type model in Scenario 2 comprises of the average simulated block model from all the realizations. Theoretically, the E- type model is identical to the kriging results in Gaussian space (Journel and Huijbregts, 1981). In Scenario 3, using SGS realizations provide a set of production scenarios that captures and assesses the uncertainty in the final pit outline, material handling in the production schedule, the overall net present value of the mining project and ore bitumen head grade.

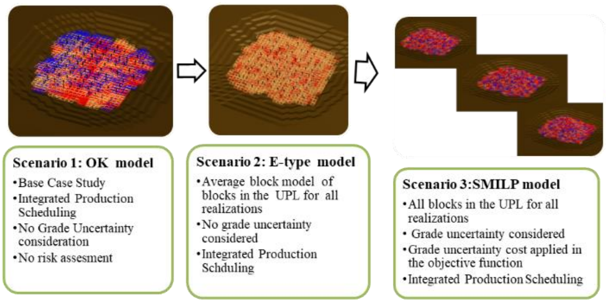


Figure 9 Implementation scenarios of case study

Table 2 Block model data of oil sands deposit for all scenarios

Parameter (Unit)	Scenarios (OK/E-type/SMILP)
Total block tonnage (Mt)	318.84
Total ore tonnages (Mt)	145.56 /165.94 / 150.26
Total OB dyke material tonnage (Mt)	39.50
Total IB dyke material tonnage (Mt)	125.16 /104.79 /120.46
Total TCS dyke material tonnage (Mt)	100.41 /111.13 /82.04
Block dimensions (m)	50 x 50 x 15
Number of blocks (#)	4476
Number of benches	6
Mine life (Years)	10

Table 3 Economic parameters for all scenarios

Parameter (Unit)	Value
Mining cost ($/tonne)	4.60
Processing cost ($/tonne)	5.03
Selling price ($/bitumen %mass)	4.50
Economic discount rate (%)	10
OB dyke material cost ($/tonne)	1.38
IB dyke material cost ($/tonne)	1.38
TCS dyke material cost ($/tonne)	0.92

Table 4 Operational material capacities and quality requirements for all scenarios

Parameter (Unit)	Max value	Min value
Mining capacity (Mt/year)	32.0	31.4
Processing capacity (Mt/year)	14.0	10.0
OB dyke material capacity (Mt/year)	3.8	1.0
IB dyke material capacity (Mt/year)	4.0	1.0
TCS dyke material capacity (Mt/year)	8.0	1.0
Ore bitumen grade (%m)	16.0	7.0
Ore fines content (%m)	30.0	0.0
IB dyke material fines content (%m)	50.0	0.0
Sand-fines ratio (%m)	6.0	0.0

Table 5 Risk parameters for all scenarios

Parameter (Unit)	Scenarios (OK/E-type/SMILP)
Number of Realizations (#)	0/0/20
Geological risk discount rate (%)	0/0/20
Penalty cost of shortage in ore production ($/tonne)	0/0/5
Penalty cost of excess in ore production ($/tonne)	0/0/10
Penalty cost of shortage in ore grade (mill) ($/%m)	0/0/2.5
Penalty cost of excess in ore grade (mill) ($/%m)	0/0/1.5

6.5 Analysis of the Optimization Results

The optimal production schedules results were obtained for the OK model, E-type model and SMILP model. All three models together with the SGS realizations were compared based on the following: cash flow, ore tonnage, waste tonnage and average bitumen grade. In this case study, three sets of experiments were conducted. The first set of experiments involves the application of the mathematical programming framework for the OK model (Scenario 1), E-type model (Scenario 2), SMILP model (Scenario 3) and each of the 20 SGS realizations. The second set of experiments consist of a comparative study on risk analysis for the given case study and, the third set of experiments include a sensitivity analysis of the SGS model based on the geological discount rate (GDR) parameter (Dimitrakopoulos and Ramazan, 2008). **Table 6** shows a summary of results obtained from the first set of experiments.

Table 6 Summary of results for OK, E-type and SGS models

Model	OK Model	E-type Model	SMILP Model
Tonnage mined (Mt)	287.60	314.23	290.87
Ore tonnage (Mt)	120.00	134.23	123.88
Waste tonnage (Mt)	147.59	167.17	145.81
Dyke material tonnage (Mt)	40.00	32.84	40.00
NPV (without dyke material cost) ($M)	1927.40	1965.52	2246.44
Dyke material cost ($M)	35.35	35.46	33.35
MIPGap (%)		5.0	

6.5.1 Comparative study: OK, E-type, SMILP and SGS Realizations Models

Figure 10 shows comparison of the cash flows from all the models and realizations. It can be seen that the cash flow from the OK model drops significantly from year 4 for the remaining periods. The cash flow of the SMILP model and the E-type model are higher and remain consistent throughout the life of mine until the ore material becomes depleted in year 10 for the SMILP model and year 11 for the E-type model. The cash flows and NPV generated by the SMILP model performs better than that of the E-type model. This is because of the penalty function introduced in the objective function to minimize the risks of not meeting the production targets in the early years of production. The NPV generated by the SMILP model was 17% better than the OK model and 14% better than the E-type model. **Figure 11** shows comparisons of the ore tonnages sent to the processing plant for all the models and realizations. Based on the figure, it is observed that the ore tonnages are approximately consistent for all the periods with the exception of the last year when the available material in the ultimate pit becomes depleted for the OK, SMILP and SGS realizations models. The E-type model indicates that more ore tonnes are being sent to the plant. This is due to the fact that the E-type model is based on the average block values of all the realizations which is sensitive to extreme values. Thus, based on the regulatory ore cutoff grade of 7 %m bitumen, majority of the bitumen grade in each mining block that falls close to the cutoff grade will tend to be classified as ore after averaging which may not be accurate. This results in over estimation of the ore tonnages but with lower average bitumen grades.

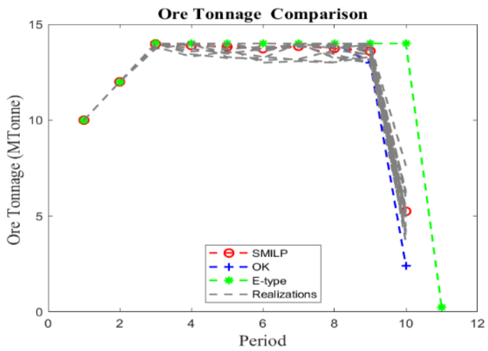


Figure 10 Ore tonnage comparisons

Figure 11 shows a comparison of the average ore bitumen grade that is fed to the mill for the three models and realizations. Based on the figure, a greater steady decline of bitumen grade could be seen for the OK model which impacts the NPV as compared to the E-type model and the SMILP model. The SMILP model generates a balanced ore bitumen grade schedule as compared to the other models throughout the mine life. This is as a result of the penalty function and geological discount rate (GDR) parameter that was introduced in the SMILP objective function to control the risk of not meeting the production targets by minimizing deviations in the early production years and deferring risk to later periods until when more geological information becomes available

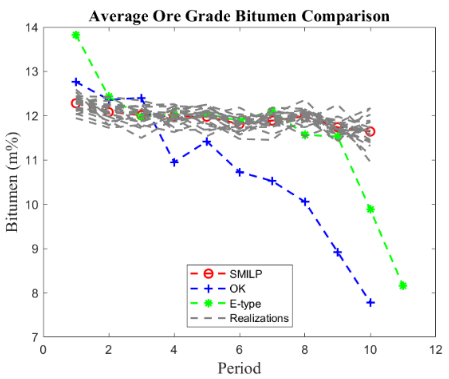


Figure 11 Ore bitumen grade comparisons

Figure 12 shows the waste tonnage comparisons that were generated as part of the production scheduling optimization. Based on the results, the E-type model generated the most total waste tonnage of 167.00 Mt as compared to the OK and SMILP models that generated 147.59 Mt and 145.81 Mt respectively. The OK and SMILP models seem to generate similar waste tonnes. However, by considering grade uncertainty in the objective function of the SMILP model, the amount of waste tonnes generated is minimized and a risk-based waste management plan can be implemented. If grade uncertainty is not considered in oil sands mine planning, there may be excess waste than the waste management plan can handle or an over-design of a waste management system to handle less waste than planned.

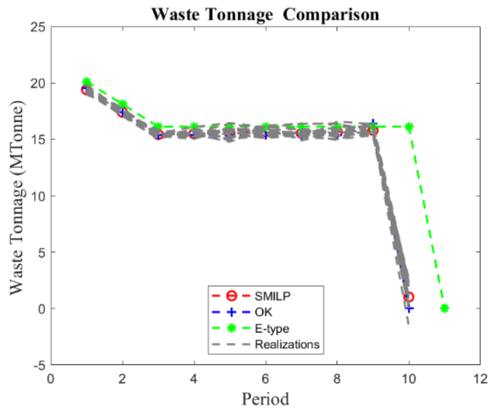


Figure 12 Waste tonnage comparisons

6.5.2 Comparative study: risk analysis

In addition to the production scheduling results, the risk profiles for cash flows, ore tonnages and ore bitumen grades were evaluated as shown in Figure 13, Figure **14** and Figure **15**. Risk profiles are calculated using the equally probable representations of the orebody from the 20 SGS realizations. The spread of bitumen grades, ore tonnages and cash flows in each realization provides an indication of uncertainty in each period according to the generated schedules.

The risk analysis was conducted by comparing the production schedules of bitumen grades, cash flows and ore tonnages for the tenth, fiftieth and ninetieth percentiles (P10, P50, and P90) with that of the three models. The results indicate that the SMILP model is similar to the P50 risk profile and therefore demonstrates that there is a strong indication of no over estimation or under estimation of the simulated NPV, cash flows, bitumen grades and ore tonnages. The scheduled results of the OK model fall below the P10 risk profile indicating that if the mine planner follows the OK model, there will be missed opportunities from following a mine plan that does not integrate grade uncertainty and is overly conservative.

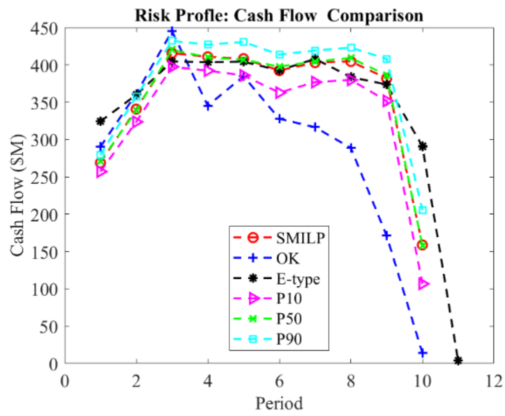


Figure 13 Risk profile for cash flow

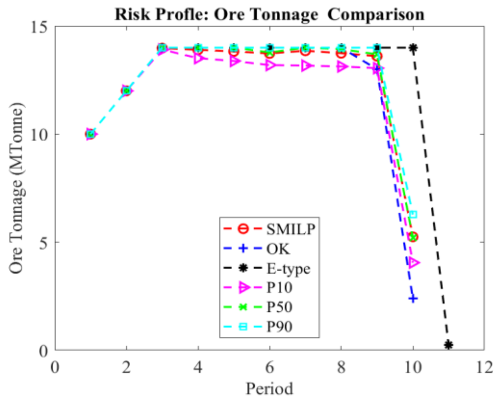


Figure 14 Risk profile for ore tonnage processed

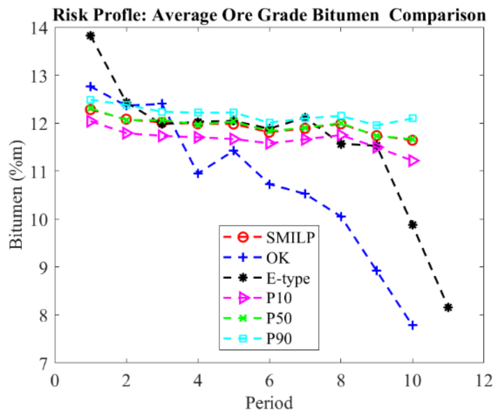


Figure 15 Risk profile for ore bitumen grade

6.5.3 Sensitivity assessment of geological discount rate (GDR) parameter

The third set of experiments were performed to conduct a sensitivity assessment of the geological discount rate (GDR) parameter used in the SMILP model. The objective of this experiment is to observe how the NPV reacts to changes in the GDR parameter. The GDR parameter is associated with the risk in the mine production schedule. A lower GDR value implies a higher risk as it is implemented as an inverse relation in the objective function. To conduct this assessment, various SMILP runs were undertaken while varying the GDR parameter and keeping all other economic and technical parameters unchanged. The results in **Figure 16** shows that as the GDR parameter increases, the NPV becomes better. This indicates that higher risk yields lower NPV and vice versa. The results also show that at GDR parameter of 30%, the optimal NPV value is attained for this case study.

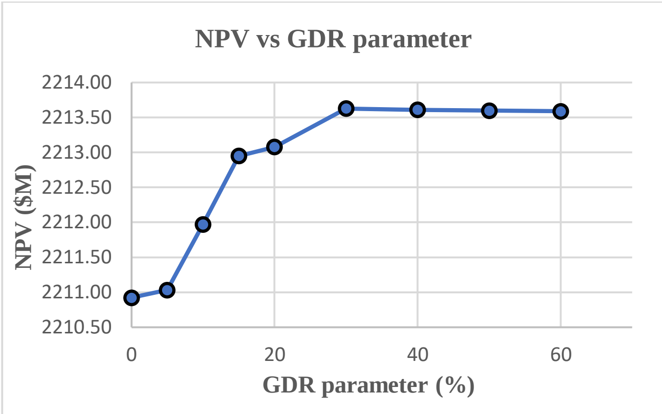


Figure 16 Sensitivity analysis of GDR parameter

7 Conclusions and Recommendations

This research has presented a Stochastic Mixed Integer Linear Programming (SMILP) framework to incorporate grade uncertainty into the optimization of integrated oil sands mine planning and waste management. Instead of using one estimated orebody model (OK model) as input to optimize the production schedule, a set of equally probable orebody realizations (20 SGS realizations) and a SMILP framework are deployed to incorporate grade uncertainty and generate a risk-based production schedule.

The stochastic optimization formulation was implemented in a Matlab/CPLEX environment. Three scenarios and three sets of experiments were conducted. The first set of experiments involve the application of the stochastic mathematical programming framework for the OK model (Scenario 1), E-type model (Scenario 2), SMILP model (Scenario 3) and each of the 20 SGS realizations. The second set of experiments consist of a comparative study on risk analysis for the given case study and, the third set of experiments include a sensitivity analysis of the SMILP model based on the geological discount rate (GDR) parameter. Based on the comparisons from the experiments, the proposed SMILP model successfully integrates grade uncertainty and generates a risk-based production schedule with stable cash flow as compared to the OK and E-type models. The OK model and E-type model do not asses the effect of grade uncertainty which impacts the NPV of the mining project. The SMILP model accounts for geological risk by deferring ore with highly uncertain grades to later years when more geological information becomes available. By deferring production risk to later years, the problem of not reaching production targets in the earlier years is minimized thus creating a smoother and stable production schedule. The results in this case study demonstrates that the SMILP model generates potential improvement in terms of the expected NPV of 17% compared to the OK model and 14% compared to the E-type model. The SMILP model also controls the geological risk compared to traditional approaches such as Ordinary Kriging and E-type that is based on single estimated orebody models. Sequential Gaussian Simulation should always be considered in mine planning so as to minimize the risk of not meeting production targets set by mine management.

Competing Interest

The authors declare that there are no competing interests.

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References

Akaike, A. and Dagdelen, K. (1999), "A strategic production scheduling method for an open pit mine", P*roceedings of the 28th Application of Computers and Operation Research in the Mineral Industry*, pp. 729-738.

Alberta Energy Regulator (2016), "Directive 082. *Operating Criteria: Resource Recovery Requirements for Oil Sands Mine and Processing Plant Operations*, pp. 1-5

Asghari, O., Soltni, F., and Bakhshandeh Amnieh, H. (2009), "*The Comparison Between Sequential Gaussian Simulation (SGS) of Choghart Ore Deposit and Geostatistical Estimation Through Ordinary Kriging*", Vol. 3, No. 1, pp. 330-341.

Badiozamani, M. M. and Askari-Nasab, H. (2014), "Integration of reclamation and tailings management in oil sands surface mine planning", *Environmental modelling & software*, Vol. 51, pp. 45-58.

Basu, S., Nandakumar, K., and Masliyah, J. H. (1996), "A Study of Oil Displacement on Model Surfaces", *Journal of Colloid and Interface Science*, Vol. 182, No. 1, pp. 82-94.

Beier, N. and Sego, D. (2008), "The oil sands tailings research facility", *Geotechnical news*, Vol. 26, No. 2, pp. 72-76.

Ben-Awuah, E. and Askari-Nasab, H. (2011), "Oil sands mine planning and waste management using mixed integer goal programming", *International Journal of Mining, Reclamation and Environment*, Vol. 25, No. 3, pp. 226-247.

Ben-Awuah, E. and Askari-Nasab, H. (2013), "Incorporating waste management into oil sands long term production planning", *Mining Technology*, Vol. 122, No. 1, pp. 33-45.

Bichard, J. A. (1987). "Oil sands composition and behaviour research. The research papers of John A. Bichard 1957-1965". Alberta Oil Sands Technology and Research Authority, Edmonton, AB, pp. 1-7.

Bowman, C. W. (1971), "The Athabasca Oil Sands Development—50 Years in Preparation", *In Origin and Refining of Petroleum*, *Advances in Chemistry*, American Chemical Society, Vol. 103, pp. 81-93.

Caccetta, L. and Giannini, L. (1986), "Optimization techniques for the open pit limit problem*",* I*n Proceedings of Proc. Australas. Inst. Min. Metall*, pp. 57-63.

Caccetta, L. and Hill, S. P. (2003), "An Application of Branch and Cut to Open Pit Mine Scheduling", *Journal of Global Optimization*, Vol. 27, No. 2, pp. 349-365.

Caccetta, L., Kelsey, P. and Giannini, L. (1998), "Open pit mine production scheduling", *In Proceedings of APCOM 98: Computer Applications in the Mineral Industries International Symposium*, pp. 65-72.

Carrigy, M. A. (1967), "Some sedimentary features of the athabasca oil sands", *Sedimentary Geology*, Vol. 1, pp. 327-352.

Chalaturnyk, R. J., Don Scott, J., and Özüm, B. (2002), "Management of oil sands tailings", *Petroleum Science and Technology*, Vol. 20, No. (9-10), pp. 1025-1046.

Conly, F. M., W Crosley, R. and Headley, J. (2011), *Characterizing sediment sources and natural hydrocarbon inputs in the lower Athabasca River, Canada*, Vol. 1, pp. 187-199.

Dagdelen, K. (1986), "Optimum open pit mine production scheduling by Lagrangian parameterization", *Proc. of the 19th APCOM*, pp. 127-142.

Dai, Q. and Chung, K. H. (1995), "Bitumen—sand interaction in oil sand processing", *Fuel*, Vol. 74, No. 12, pp. 1858-1864.

Denby, B. and Schofield, D. (1995), "The use of genetic algorithms in underground mine scheduling", *In Proceedings of Proceedings of the 25th international symposium application of computers mineral industries*, pp. 389-394.

Denby, B., Schofield, D., and Bradford, S. (1991), "Neural network applications in mining engineering", *Department of Mineral Resources Engineering Magazine, University of Nottingham*, pp. 13-23.

Deutsch, C. V. and Journel, A. G. (1992), *GSLIB: Geostatistical Software Library and User's Guide. Hauptbd*, Oxford university press, pp. 169-181.

Dimitrakopoulos, R. and Ramazan, S. (2003), "Managing risk and waste mining in long-term production scheduling of open pit mine", *In Proceedings of 2003 SME Annual Meeting & Exhibit*, pp. 24-26.

Dimitrakopoulos, R. and Ramazan, S. (2008), "Stochastic integer programming for optimising long term production schedules of open pit mines: methods, application and value of stochastic solutions", *Mining Technology*, Vol. 117, No. 4, pp. 155-160.

Eckert, W. F., Masliyah, J. H., Gray, M. R., and Fedorak, P. M. (1996), "Prediction of sedimentation and consolidation of fine tails", *AIChE Journal*, Vol. 42, No. 4, pp. 960-972.

Gershon, M. E. (1983), "Mine scheduling optimization with mixed integer programming", *Min. Eng.(Littleton, Colo.);(United States)*, Vol. 35, No. 4.

Gleixner, A. M. (2009), "Solving large-scale open pit mining production scheduling problems by integer programming", *Thesis*, Institute for Mathematics, Faculty II: Mathematics and Natural Sciences, Technical Unversity of Berlin, pp. 1-127.

Godoy, M. and Dimitrakopoulos, R. (2004), "Managing risk and waste mining in long-term production scheduling of open-pit mines", *SME transactions*, Vol. 316, No. 3, pp. 43-50.

Hall, A. C., Collins, S. H., and Melrose, J. C. (1982), *Stability of Aqueous Wetting Films in Athabasca Tar Sands*, *Society of Petroleum Engineers Journal*, Vol. 23, No. 02, pp.249-258.

Hein, F. J. and Cotterill, D. K. (2006), "The Athabasca Oil Sands — A Regional Geological Perspective, Fort McMurray Area, Alberta, Canada", *Natural Resources Research*, Vol. 15, No. 2, pp. 85-102.

Hooshiar, A., Uhlik, P., Ivey, D. G., Liu, Q., and Etsell, T. H. (2012), "Clay minerals in nonaqueous extraction of bitumen from Alberta oil sands: Part 2. Characterization of clay minerals", *Fuel Processing Technology*, Vol. 96, pp. 183-194.

Isaaks, E. H. and Srivastava, R. M. (1989), *An introduction to applied geostatistics*, Oxford university press, New York, pp. 1-587.

Johnson, T. (1988), "Application of the Max Flow algorithm to ultimate pit design", *Engineering design: better results through operations research methods*, pp. 518-531.

Johnson, T. B. (1968), "*Optimum open pit mine production scheduling*", California University Berkeley Operations Research Center, Paper Document Number ORC-68-11.

Journel, A. and Isaaks, E. (1984), "Conditional indicator simulation: application to a Saskatchewan uranium deposit", *Journal of the International Association for Mathematical Geology*, Vol. 16, No. 7, pp. 685-718.

Journel, A. G. and Huijbregts, C. J. (1978), *Mining geostatistics*, Academic press, pp. 1-600.

Kalantari, S., Ben-Awuah, E., and Askari-Nasab, H. (2013), "Towards an integrated oil sands mine plan and composite tailings plan", *International Journal of Mining, Reclamation and Environment*, Vol. 27, No. 2, pp. 103-126.

Kaminsky, H. (2008), "Characterization of an Oil Sand Ore and Process Streams", *PhD Thesis*, University of Alberta, pp. 1-368.

Kaminsky, H., H. Etsell, T., G. Ivey, D., and Omotoso, O. (2008), "*Characterization of Heavy Minerals in Alberta Oil Sands*", Vol. 21, pp. 264-271.

Kasongo, T., Zhou, Z., Xu, Z., and Masliyah, J. (2010), "Effect of clays and calcium ions on bitumen extraction from athabasca oil sands using flotation", *The Canadian Journal of Chemical Engineering*, Vol. 78, No. 4, pp. 674-681.

Kasperski, K. (1992), "A review of properties and treatment of oil sands tailings", *AOSTRA Journal of Research*, Vol. 8, pp. 11-11.

Kotlyar, L. S., Ripmeester, J. A., Sparks, B. D., and Montgomery, D. S. (1988), "Characterization of organic-rich solids fractions isolated from Athabasca oil sand using a cold water agitation test", *Fuel*, Vol. 67, No. 2, pp. 221-226.

Koushavand, B., Askari-Nasab, H., and Deutsch, C. V. (2014), "A linear programming model for long-term mine planning in the presence of grade uncertainty and a stockpile", *International Journal of Mining Science and Technology*, Vol. 24, No. 4, pp. 451-459.

Krige, D. (1951), "A Statistical Approach to Some Basic Mine Valuation Problems on the Witwatersrand", *Journal of the Chemical, Metallurgical and Mining Society of South Africa*, Vol. 52, No. 6, pp. 119-139.

Lerchs, H. and Grossmann, I. (1965), "*Optimum Design of Open Pit Mines*", Transactions of the Canadian Institute of Mining Bulletin, Vol. 68, pp. 17-24.

Liu, J., Xu, Z., and Masliyah, J. (2008), "Interaction between Bitumen and Fines in Oil Sands Extraction System: Implication to Bitumen Recovery", *The Canadian Journal of Chemical Engineering*, Vol. 82, No. 4, pp. 655-666.

Long, J., Drelich, J., Xu, Z., and Masliyah Jacob, H. (2008), "Effect of Operating Temperature on Water-Based Oil Sands Processing", *The Canadian Journal of Chemical Engineering*, Vol. 85, No. 5, pp. 726-738.

MacKinnon, M. (1989), "Development of the tailings pond at Syncrude's oil sands plant: 1978–1987", *AOSTRA Journal of Research*, Vol. 5, No. 2, pp. 109-133.

Masliyah, J., Zhou, Z. A., Xu, Z., Czarnecki, J., and Hamza, H. (2004), "Understanding Water-Based Bitumen Extraction From Athabasca Oil Sands", *The Canadian Journal of Chemical Engineering*, Vol. 82, pp. 628-654.

Masliyah, J. H., Czarnecki, J., and Xu, Z. (2011), *Handbook on theory and practice of bitumen recovery from Athabasca oil sands - Volume 1: Theoretical basis*, Canada: Kingsley Knowledge Publishing, pp.211-250.

Matheron, G. (1971), "The theory of regionalised variables and its applications", *Les Cahiers du Centre de Morphologie Mathématique*, Vol. 5, pp. 1-211.

McFadyen, D. (2009), "Directive 074: Tailings Performance Criteria and Requirements for Oil Sands Mining Schemes", *Energy Resources Conservation Board, Calgary, AB, Canada*, pp. 1-14.

Meagher, C., Dimitrakopoulos, R., and Avis, D. (2014), "Optimized open pit mine design, pushbacks and the gap problem—a review", *Journal of Mining Science*, Vol. 50, No. 3, pp. 508-526.

Mikula, R., Munoz, V., Kasperski, K., Omotoso, O., and Sheeran, D. (1998), "Commercial implementation of a dry landscape oil sands tailings reclamation option: consolidated tailings", *In Proceedings of Proceedings 7th UNITAR Conference*, pp. 907-921.

Mikula, R. J., Omotoso, O., and Kasperski, K. L. (2008), *The chemistry of oil sands treatment: Production to treatment*. Edmonton, AB: University of Alberta Geotechnical Center and Oil Sands Tailing Research Facility, pp. 23-33 .

Mitchell, J. E. (2002). Branch-and-cut algorithms for combinatorial optimization problems. *Handbook of applied optimization*, pp. 65-77.

Mossop, G. D. (1980). Geology of the Athabasca Oil Sands. *Science*, Vol. 207 (4427), pp.145- 158.

Onur, A. and Dowd, P. (1993), *Open-pit optimization - Part 2: Production scheduling and inclusion of roadways.* Vol. 102, pp. 105-113.

Osacký, M., Geramian, M., Uhlík, P., Čaplovičová, M., Danková, Z., Pálková, H., Vítková, M., Kováčová, M., Ivey, D. G., Liu, Q., and Etsell, T. H. (2017), "Mineralogy and Surface Chemistry of Alberta Oil Sands: Relevance to Nonaqueous Solvent Bitumen Extraction", *Energy and Fuels*, Vol. 31, No. 9, pp. 8910-8924.

Osanloo, M., Gholamnejad, J., and Karimi, B. (2008), "Long-term open pit mine production planning: A review of models and algorithms", *International Journal of Mining, Reclamation and Environment,* Vol. 22, No. 1, pp. 3-35.

Picard, J.-C. (1976), "Maximal closure of a graph and applications to combinatorial problems", *Management science*, Vol. 22, No. 11, pp. 1268-1272.

Ramazan, S. (2007), "The new fundamental tree algorithm for production scheduling of open pit mines" *European Journal of Operational Research*, Vol. 177, No. 2, pp. 1153-1166.

Ramazan, S., Dagdelen, K., and Johnson, T. (2005), "Fundamental tree algorithm in optimising production scheduling for open pit mine design", *Mining Technology*, Vol. 114, No. 1, pp. 45-54.

Ramazan, S. and Dimitrakopoulos, R. (2018), "Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation", *In Advances in Applied Strategic Mine Planning*, Springer, pp. 139-153.

Rambert, F. (2005), Introduction to Mining Geostatistics, *In Proceedings of 67th EAGE Conference and Exhibition-Workshops*, pp. cp-140

Ravenscroft, P. J. (1992), "Recoverable reserve estimation by conditional simulation", *Geological Society, London, Special Publications*, Vol. 63, No. 1, pp. 289-298.

Rossi, M. E. and Deutsch, C. V. (2013), *Mineral resource estimation*, Springer Science & Business Media, pp. 1-327.

Royle, A., Clausen, F., and Frederiksen, P. (1981), Practical universal kriging and automatic contouring. *Geoprocessing*, Vol. 1, pp. 377-394.

Samal, A. R., Mohanty, M. K., and Fifarek, R. H. (2008), "Backward elimination procedure for a predictive model of gold concentration", *Journal of Geochemical Exploration*, Vol. 97, No. (2-3), pp. 69-82.

Sanford, E. C. (1983), Processibility of athabasca oil sand: Interrelationship between oil sand fine solids, process aids, mechanical energy and oil sand age after mining. *The Canadian Journal of Chemical Engineering*, Vol. 61, No. 4, pp. 554-567.

Schofield, N. and Rolley, P. (1997), Optimisation of ore selection in mining: Method and Case Studies. *In Proceedings of Proceedings Third International Mining Geology Conference*, pp. 105-118

Sheeran, D. (1993), "An improved understanding of fine tailings structure and behaviour", *In Proceedings of Proceedings fine tails symposium, oil sands–our petroleum future conference*, Edmonton: AOSTRA Publications, pp. 19-25.

Sinclair, A. J. and Blackwell, G. H. (2002), *Applied Mineral Inventory Estimation*, Cambridge University Press, Cambridge, pp. 59-66

Soane, D., Ware, W., Mahoney, R., and Kincaid, K. (2010), "*Oil sands tailings treatment via surface modifications of solids with polymers*", *In Proceedings of Proceedings of the 2nd International Oil Sands Tailings Conference*, pp. 5-8.

Tabesh, M. and Askari-Nasab, H. (2011), "Two-stage clustering algorithm for block aggregation in open pit mines", *Mining Technology*, Vol. 120, No. 3, pp. 158-169.

Takamura, K. (1982), "Microscopic structure of athabasca oil sand", *The Canadian Journal of Chemical Engineering*, Vol. 60, No. 4, pp. 538-545.

Vann, J., Bertoli, O., and Jackson, S. (2002), "An overview of geostatistical simulation for quantifying risk", *In Proceedings of Proceedings of Geostatistical Association of Australasia Symposium,* Quantifying Risk and Error, pp. 131-151.

Wallwork, V., Xu, Z., and Masliyah, J. (2008), "Bitumen Recovery with Oily Air Bubbles", *The Canadian Journal of Chemical Engineering*, Vol. 81, No. 5, pp. 993-997.

Wik, S., Sparks, B. D., Ng, S., Tu, Y., Li, Z., Chung, K. H., and Kotlyar, L. S. (2008). Effect of process water chemistry and particulate mineralogy on model oilsands separation using a warm slurry extraction process simulation. *Fuel*, Vol. 87, No. 7, pp. 1394-1412.

Zhou, Z. A., Xu, Z., Masliyah, J. H. and Czarnecki, J. (1999), "Coagulation of bitumen with fine silica in model systems", *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, Vol. 148, No. 3, pp. 199-211.

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