Comparison of Analytical and Numerical Water Influx Models in Bottom Water Reservoir

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Abstract

Almost all hydrocarbon reservoirs are bounded by water-bearing rocks called aquifers. In reservoir modelling and simulation, there are more uncertainties attached to aquifer modelling than any other subject. Water influx models are mathematical models that help to simulate and predict aquifer performance. In the quest to analyse the cumulative water influx in a bottom-water drive reservoir for an analytical and a numerical model, the following water influx models were used in both cases: Van Everdingen-Hurst model, Carter Tracy model and Fetkovich model. The analytical approach was programmed in MATLAB while the numerical approach was achieved by using specific keywords in the ECLIPSE 100 software. The Van Everdingen model is not found in the ECLIPSE 100 software, therefore, the Carter Tracy model which is an approximation of Van Everdingen model, was used for the comparison since it can be modelled using the ECLIPSE 100 software. After the cumulative water influx of all the models for both cases (analytical and numerical) were calculated, the numerical approach yielded more realistic field results because of the degree of heterogeneity of the parameters – porosity and permeability incorporated into the modelling process. For Carter Tracy model, the results obtained from the numerical and analytical approach after a production period of 720 days appeared to be identical with minimal approximation error. With the Fetkovich model, there was a considerable difference in approximation error between the results from the two approaches after the same period of production of 720 days, but after 1460 days of production, the results of the two methods (analytical and numerical) became identical with minimal approximation error.

Keywords: Water Influx, Analytical Aquifer Modelling, Numerical Modelling, Bottom Water Reservoir

1 Introduction

Nearly all hydrocarbon water drive reservoirs are surrounded by water-bearing rocks called aquifers (Ahmed, 2001). During production, the reservoir pressure declines and this brings about a pressure differential between the reservoir and its surrounding aquifer. In response to the pressure drop caused by production, and when there is a hydraulic connection between the reservoir and aquifer and with the aquifer having a larger size and permeability, water in the aquifer starts to expand and then flows into the reservoir to balance the pressure decline. Other than the expansion of water in the aquifer, there are several other factors which cause water influx into the reservoir. These include expansion of other accumulations (known and unknown) of hydrocarbon in aquifer rock, aquifer rock compressibility and artesian flow, especially when the water bearing formation is located at a structurally higher position than the pay zone (Osei, 2020). Water influx models are mathematical representations of the aquifer which are used to simulate and predict both the performance and the cumulative water influx history of a reservoir (Okotie and Ikporo, 2019).

However, it should be noted that in reservoir engineering, there is a vast uncertainty in predicting water influxes from aquifers than any other subject. This high level of uncertainty is attributed to the fact that, aquifers are not directly explored in order to obtain their underlying properties such porosity, permeability and the properties of the fluid they contain. Instead, these necessary properties of the aquifer are inferred from drilling to the reservoir. Areal continuity and geometry of the aquifer are two parameters of the aquifer that are highly unpredictable. Most of the proposed aquifer models have been developed based on assumptions on the aquifer (Anon, 2020). The characteristic nature of aquifers poses great uncertainties which makes it essential to match historical reservoir performance data in order to evaluate constants that represent aquifer properties when modelling the aquifer, since these are rarely known from exploration-development drilling with sufficient accuracy for direct application. However, an alternative approach in determining historical water influx is the use of material balance equation (McEwen, 1962). This approach is feasible if the original oil in place is known from pore volume estimates. Future water influx can then be predicted from the constants evaluated from the material balance equation. Both past and recent studies indicate little comprehensive work done in comparing analytical and numerical water influx models. This component is key in evaluating the effectiveness and accuracy of the existing water influx models. In this regard, this work seeks to compare the analytical water influx models to numerical simulation (Eclipse) for forecasting future water influx in bottom water reservoirs taking into consideration water influx models.

A survey of literature shows that a tremendous amount of research has been conducted on water influx in oil reservoirs. As a result, several models have been proposed to estimate water influx based on assumptions on the aquifer properties. However, these water influx models have barely been compared to pinpoint the variations amongst them, hence, this research sought to evaluate these disparities.

1.1 Modelling the Water Inflow into a Reservoir

Reservoirs normally have aquifers which provide pressure support to the reservoir (Maganga, 2017). The reservoir and the aquifer form a hydraulic system and a pressure decline accompanying production results in water encroachment into these reservoirs. The importance of this water movement is derived from the significant dependence of production rate on reservoir pressure, which in turn depends on the rate of encroaching water. For this reason, a successful simulation study is only possible if the complete system, reservoir and aquifer, is taken into consideration and not only the hydrocarbon bearing part. Fundamentally there are two possibilities to model the water inflow into a reservoir. These are, representing the aquifer by a grid (numerical) model, and using analytical models.

1.1.1 Gridded (Numerical) Aquifer Models

At the beginning of a simulation project, unlike the geological and petro-physical properties of the productive zones, that of the aquifer is usually unknown. This is then determined by matching the reservoir pressure in order to estimate the size, the porosity, the permeability and their distributions around the productive area. In the absence of other possibility, the aquifer is regarded isotropic in areal extension, i.e. the permeability is independent of its direction. The history matching process (Abraham, 2009) involves resizing and parameterisation of the aquifer without changing its productive areas in order to have the detailed model of the aquifer.

1.1.2 Analytical Aquifer Modelling

To establish aquifer models, two different approaches can be used, the first approach entails establishing a model based on idealized mathematical models, and these models are idealized as long as they assume homogeneous reservoir properties like uniform porosity, permeability, etc. But these models are also idealized in another way, regarding the reservoir and flow geometry. This idealization is expressed either by radial or linear models. The second approach is to develop aquifer behaviour models that are based on a direct integration of field data. This approach leads to models without idealized assumptions regarding homogeneities and geometries. In 1936 Schilthuis published a model according to the first approach described above. Schilthuis developed a model for steady-state water influx behaviour, this means that this model is applicable if the aquifer is of such an extent that water influx to the reservoir does not alter the aquifer pressure observably. This would correspond to an aquifer of infinite dimensions since the size of an aquifer is usually limited, the method is only applicable in a few cases.

2 Resources and Methods

2.1 Introduction

The current reservoir characterisation, modelling, advanced well logging and interpretation methods, including production surveying and monitoring systems deliver more detailed information about the reservoir in context. The improvement in quality and quantity of data generally reduces the attempt of the same process, but all of these methods usually are not applied to the aquifer of the reservoir. Considering that the aquifer is frequently not covered by modern reservoir modelling techniques, great uncertainty regarding the characterisation and parameters are the result.

2.2 Water Influx Models

Water influx models are mathematical models that simulate and predict cumulative water influx into the reservoir. However, all these models are developed based on assumptions of the aquifer properties. For the scope of this project, the water influx models considered under study include the Van Everdingen-Hurst unsteady-state model, the Carter-Tracy unsteady-state model and Fetkovich model. Table 1 presents the input parameters used in the computations of both the analytical and numerical models in MATLAB and ECLIPSE 100, respectively. The same reservoir data is used for both numerical and analytical models because the project seeks to compare the behaviour of water influx under the same set of reservoir conditions.

Table 1. Input Parameters for the Analytical and Numerical Computations

Parameter	Values	Units
Radius, r_e (reservoir)	2000	ft
Radius, r_a (aquifer)	∞	ft
Height, h (reservoir)	20	ft
Height, h (aquifer)	25	ft
Permeability, k (reservoir)	50	mD
Permeability, k (aquifer)	100	mD
Porosity, φ (reservoir)	15	%
Porosity, φ (aquifer)	20	%
Viscosity, μ_w (reservoir)	0.5	cP
Viscosity, μ_w (aquifer)	0.8	cP
p_i-p	10	psi
Total compressibility coefficient, (c_t)	0.000001	psi^{-1}
Encroachment angle (θ)	360	

2.2.1 Van Everdingen-Hurst Unsteady-State

Van Everdingen and Hurst (1949) developed solutions to the dimensionless diffusivity equation for two reservoir-aquifer boundary conditions. These are the constant terminal rate, and the constant terminal pressure. The constant-terminal-rate boundary condition is such that, the rate of water influx is assumed constant for a given period and the pressure drop at the reservoir-aquifer boundary is calculated. For the constant-terminal-pressure boundary condition, the boundary pressure is assumed to be constant over a specified period while the water influx rate is calculated.

This model considers the edge-water-drive system (radial), bottom-water-drive system and linear-flow systems. The radial model describes a reservoir as a cylindrical object which is surrounded by an aquifer.

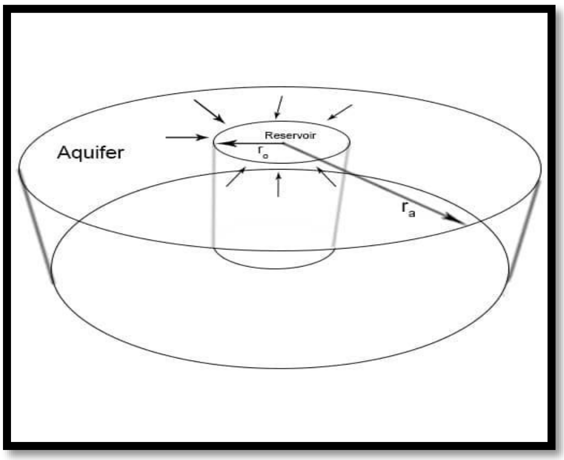

Fig. 1 Radial Aquifer Model (Ahmed, 2001)

The Van Everdingen and Hurst model is the most sophisticated amongst these models. However, it is realistic but cumbersome in nature. Charts or labels had to be consulted repeatedly to execute a single calculation. Carter-Tracy and Fetkovich later proposed models which were devoid of charts and tables. These models, however, are only approximations to and simplifications of the Van Everdingen and Hurst model.

Edge-Water Drive

Figure 1 is a generalised radial flow system for an edge-water drive reservoir. The inner boundary is considered as the interface between the reservoir and the aquifer. The flow vectors in this case are horizontal, with water encroaching across a cylindrical plane encircling the reservoir. The mathematical expression that relates the water influx (We) to the dimensionless water influx (W_{eD}), dimensionless time (t_D) and dimensionless radius (r_D), for an edge water drive reservoir is developed by generalising the diffusivity equation in cases where the flow of water is radial. The solutions obtained in this paper were derived for cases of both bounded and infinite aquifers. Therefore, the water influx for the edge-water drive is given as equation (1):

$$W_e = B\Delta p\, W_{eD} \tag{1}$$

With the water influx constant, B (bbl/psi) is represented in equation (2) as:

$$B = 1.119\varphi c_t r_e^2 h \tag{2}$$

Where, We = cumulative water influx, bbl

B = water influx constant, bbl/psi

Δp = pressure drop at the boundary, psi

W_{eD} = dimensionless water influx

Bottom-Water Drive

Van Everdingen-Hurst also proposed a solution to the radial diffusivity equation. This solution however, is the most rigorous amongst them all. The flow vectors for this model is predominantly vertical with its water encroachment occurring across a horizontal circular plane representing the oil/water contact. The diffusivity equation is then modified to account for the vertical flow by adding another term in the equation to give equation (3).

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + Fk\frac{\partial^2 p}{\partial z^2} = \frac{\mu\varphi c}{k}\frac{\partial p}{\partial t} \tag{3}$$

Where F_k is the vertical to horizontal permeability, or;

$$F_K = {}^{kv}\!/_{kh} \tag{4}$$

A newly introduced dimensionless variable z_D is represented by equation (5):

$$z_D = \frac{h}{r_e\sqrt{F_K}} \tag{5}$$

Therefore, equation (1) is also used to calculate the water influx for the bottom-water drive. However, the values of W_{eD} in this case are different from those of the van Everdingen-Hurst's edge-water drive model. This is because W_{eD} for the bottom-water drive is a function of the vertical permeability. The values of W_{eD} are tabulated as a function of r_D, t_D, and z_D. The solution process for bottom-water influx problem is similar to that of the edge-water influx.

2.2.2 Carter-Tracy Water Influx Model

Carter and Tracy (1960) proposed a much simpler calculation technique that does not require superposition. The author's method allows for direct alculation of water influx. The main difference between the Van Everdingen-Hurst technique and the Carter Tracy's technique is that the latter assumes constant water influx rates over each finite time interval. For the Carter-Tracy technique, the cumulative water influx at any time t_n can be calculated directly from the previous value obtained at t_{n-1}, as shown in equation (6):

$$(W_e)_n = (W_e)_{n-1} + [(t_D)_n - (t_D)_{n-1}]\left[\frac{B\Delta p_n - (W_e)_{n-1}(p'_D)_n}{(p_D)_n - (t_D)_{n-1}(p'_D)_n}\right] \tag{6}$$

Where, B = the van Everdingen-Hurst water influx constant as defined by equation (2)

t_D = dimensionless time

n = current time step

n - 1= previous time step

ΔP_n = total pressure drop, $p_i - p_n$ psi

p_D = dimensionless pressure

p'_D = dimensionless pressure derivative

Values of the dimensionless pressure p_D as a function of t_D and r_D are already tabulated. Equation (7) presents an approximation of p_D for an infinite-acting aquifer:

$$p_D = \frac{370.529\sqrt{t_D} + 137.582 t_D + 5.69549 (t_D)^{1.5}}{328.834 + 265.488\sqrt{t_D} + 45.2157 t_D + (t_D)^{1.5}} \quad (7)$$

The dimensionless pressure derivative can then be approximated as equation (8):

$$p'_D = \frac{E}{F} \quad (8)$$

Where,

$$E = 716.441 + 46.7984 (t_D)^{0.5} + 270.038 t_D + 71.0098 (t_D)^{1.5} \quad (9)$$

$$F = 1296.86 (t_D)^{0.5} + 1204.73 t_D + 618.618 (t_D)^{1.5} + 538.072 (t_D)^2 + 142.41 (t_D)^{2.5} \quad (10)$$

Therefore, when $t_D > 100$, P_D is approximated by equation (11):

$$p'_D = 0.5 \left[\ln (t_D + 0.80907) \right] \quad (11)$$

And the derivative as given by:

$$p'_D = 1/(2t_D) \quad (12)$$

It should be noted that the Carter-Tracy method is not an exact solution to the diffusivity equation and should be considered as an approximation.

2.2.3 Fetkovich's Method

Fetkovich (1971) developed a method of describing the approximate water influx behaviour of a finite aquifer for radial and linear geometries. The Fetkovich theory is much simpler compared to other models. This method does not require the use of superposition. Therefore, the application is much easier. Fetkovich's model is based on the assumption that, the water influx in a finite aquifer reservoir can be adequately described by the concept of productivity index. That is to say that the rate of water influx is directly proportional to the pressure drop between the average aquifer pressure and the pressure at the reservoir-aquifer boundary. Fetkovich came up with an integration to give the following form:

$$W_e = \frac{W_{ei}}{p_i}(p_i - p_r)\exp\left(\frac{-jp_i t}{W_{ei}}\right) \quad (13)$$

Where, W_e = cumulative water influx, bbl

p_r = reservoir pressure

t = time, days

Since Equation (13) was derived for a constant inner boundary pressure, it tends to have no practical applications. Nonetheless, a superposition technique can be applied in order to use it for a continuously varying boundary pressure. In place of the superposition method, Fetkovich's method can be used. This method suggests that, the pressure history at the reservoir boundary can be divided into a finite number of time intervals. And each incremental water influx during the nth interval is given as equation (14):

$$(\Delta W_e)_n = \frac{W_{ei}}{P_i}\left[(\bar{P}_a)_{n-1} - (\bar{P}_r)_n\right]\left[1 - \exp\left(\frac{-jp_i \Delta t_n}{W_{ei}}\right)\right] \quad (14)$$

Where, $(\bar{p}_a)_{n-1}$ is the average aquifer pressure at the end of the previous time step. This average pressure is calculated from equation (15) as:

$$(\bar{p}_a)_{n-1} = p_i\left(1 - \frac{(W_e)_{n-1}}{W_{ei}}\right) \quad (15)$$

The average reservoir boundary pressure $(\bar{p}_r)_n$ is estimated from equation (16) as:

$$(\bar{p}_r)_n = \frac{(p_r)_n + (p_r)_{n-1}}{2} \quad (16)$$

2.3 Aquifer Modelling Facilities in Eclipse

There are several ways to model an aquifer using the Eclipse software. These include numerical, analytical aquifer (such as Fetkovich and Carter-

Tracy aquifers), constant flux aquifer and constant head aquifer (Schlumberger, 2015).

2.3.1 Numerical Modelling Facilities in Eclipse

Numerical aquifer is created within a simulation grid by highlighting one-dimension row of cells using the keyword AQUNUM. The keyword AQUNUM contains information about the aquifer properties such as length, cross-section area, porosity, permeability, initial pressure, depth, PVT, and saturation table numbers. A non-neighbour connections (NNCs) to the reservoir faces are specified by using the keyword AQUCON. Both keywords AQUNUM and AQUCON are specified in the grid section. The keyword AQUDIM is used to specify the dimensions in the RUNSPEC section (Schlumberger, 2015).

2.3.2 Analytical Aquifer

Analytical aquifers can also be modelled in the Eclipse software. It is created with the help of source terms in the reservoir grid cells by using the keywords such as, AQUCT (for Carter-Tracy aquifer) or AQUFET/AQUFETP (for Fetkovich aquifer). The aquifer is connected to the reservoir by using the keyword AQUANCON. Both keywords AQUCT, AQUFET and AQUANCON are specified in the solution section. The dimensions of the aquifer are specified by using the keyword AQUDIMS in the RUNSPEC. Figure 2 presents some of the computations for the aquifer models.

2.3.3 Constant Flux Aquifer

A constant flux aquifer in the Eclipse 100 software is defined by using the keyword AQUFLUX. It is connected to the reservoir grid by non-neighbour connection defined in the keyword AQUANCON. Both keywords AQUFLUX and AQUANCON are specified in SOLUTION section. The user directly specifies the flow rate of flux aquifer. The negative rate means the flux is out of the reservoir. In RUNSPEC section, flux aquifer is treated the same as analytical aquifer.

2.3.4 Constant Head Aquifer

A constant head aquifer is defined by the keyword AQUCHWAT for water aquifer and AQUCHGAS for gas aquifer. The aquifer connection to the reservoir faces is made by using the keyword AQUANCON or AQANCONL. The difference between the latter two keywords is the connection to a global cell or local grid cell. The keywords AQUCHWAT, AQUCHGAS, AQANCON and AQANCONL are specified in SOLUTION section. In RUNSPEC section, the keyword AQUDIMS should be set to define the parameters NANAQU (maximum number of analytical aquifers) and NCAMAX (maximum number of grid blocks connected to aquifer) for the facility to function.

```
AQUDIMS
1   1   2   62   1   450/

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GRID
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NOECHO

DX
 90*1000 /
DY
 90*1000 /
DZ
 90*50 /

TOPS
  30*5000 30*5050 30*5100   /

PERMX
  90*200 /
PERMY
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Fig. 2 Aquifer Model Computation for the Eclipse Software.

3 Results and Discussion

3.1 Analytical Aquifer Modelling

From the data provided, the cumulative water production (W_e) for the various water influx models is determined using their respective formulae programmed in MATLAB (analytical). Table 1 shows the input parameters for the computations.

3.1.1 Carter Tracy Model (Numerical) vs. Van Everdingen Model (Analytical)

The Carter Tracy numerical and Van Everdingen analytical models yielded cumulative water influxes of about 176 666.80 bbl and 225 360.54 bbl, respectively. Although the Van Everdingen-Hurst method gives the exact solution to the radial diffusivity equation, the solution has to be superimposed. The superposition normally entails tedious calculations which is time consuming. However, Carter-Tracy's method of calculating water influx is direct and does not involve superposition yet it yields similar results as that of the Van Everdingen-Hurst's. Carter Tracy's method is therefore considered as an approximation of the Van Everdingen-Hurst's method. Figure 3 below presents graphical comparison of the Carter Tracy Model (Numerical) and Van Everdingen water influx model (Analytical).

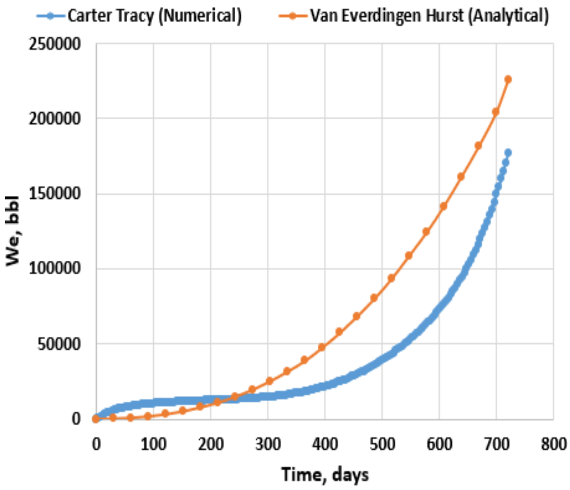

Fig. 3 Graphical Comparison of the Carter Tracy Model (Numerical) and Van Everdingen Water Influx Model (Analytical)

3.1.2 Fetkovich Model (Analytical)

Fetkovich's model (analytical) after a period of production of 1460 days experienced a cumulative water influx of 26 635 592 bbl of water while at 720 days its water influx was calculated to be 8 552 092 bbl. The difference could be attributed to the fact that the cumulative water influx in the mathematical equation proposed by Fetkovich is a direct function of pressure drop across the reservoir and this relates to time (t) exponentially. This implies that, the longer the production period from the reservoir, the higher the rate of water influx. Fig. 4 presents a graphical representation of Fetkovich's water influx model (Analytical) for a period of production of 720 days.

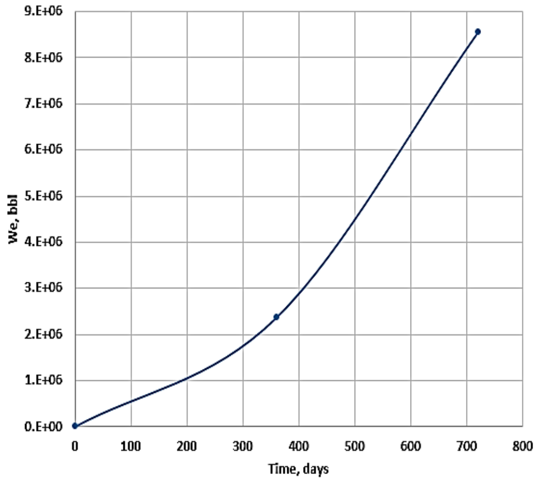

Fig. 4 Graphical Presentation of Fetkovich's model (Analytical)

3.2 Numerical Aquifer Models

From the data provided in Table 1, the cumulative water production (W_e) for the various water influx models is determined using Eclipse software. This is accomplished by modelling a reservoir and including an aquifer matching the properties of the subject model. The results from the simulations are displayed in the eclipse results viewer.

3.2.1 Carter Tracy Model (Numerical)

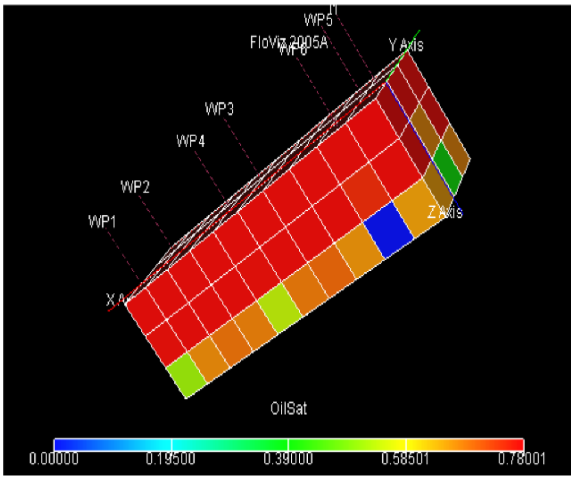

Fig. 5 Visual Presentation of a Reservoir for Carter Tracy Model

Figure 5 presents a visual interpretation of a reservoir with a Cater Tracy aquifer whereas Fig. 6 shows a graphical representation of the numerical Carter Tracy water influx results for a period of production of 720 days.

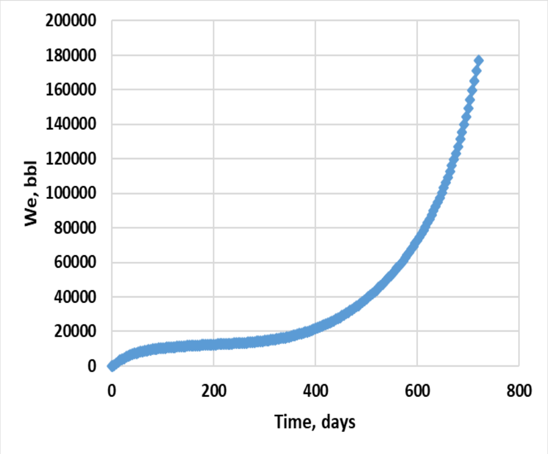

Fig. 6 Cumulative Water Influx for Carter Tracy Model (Numerical)

3.2.1 Fetkovich Model (Numerical)

Fig. 7 presents a visual representation of the reservoir with the blue surface representing the aquifer with specified Fetkovich properties.

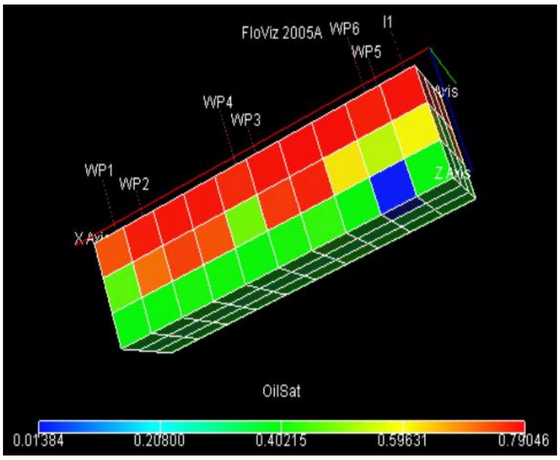

Fig. 7 Visual Presentation of a Reservoir for Fetkovich Model (Numerical)

From the results obtained after the simulation with the Eclipse 100 software, it was observed that the Field Water Production Total (FWPT) or Cumulative Water Influx for the Fetkovich aquifer model was 3 086 579 bbls within a period of 720 days. Other field parameters such as, Field Water Production Rate (FWPR) and Field Production Rate (FPR) yielded attainable values. Figure 8 shows a graphical representation of Fetkovich water influx model (Numerical) for a period of production of 720 days.

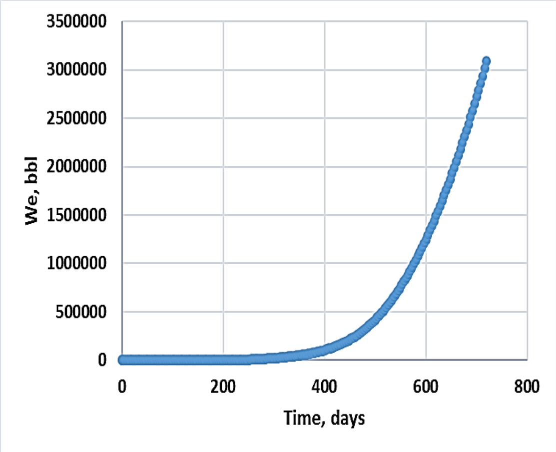

Fig. 8 Cumulative Water Influx for Fetkovich Model (Numerical)

3.3 Analytical vs. Numerical Aquifer Modelling

Figure 9 as well as Fig. 10 presents detailed comparison of the cumulative water influx between analytical and numerical modelling methods for the Carter Tracy and Fetkovich aquifer models, respectively with the help of history matching. A successful simulation study is only possible if the complete system, reservoir and aquifer, is taken into consideration. But the analytical method is considered to be idealized as long as they assume homogeneous reservoir properties like uniform porosity, permeability, reservoir, flow geometry etc. The numerical method is considered a more realistic approach since it considers certain heterogeneity in reservoir properties. Hence, the numerical approach is more likely to yield a more accurate water influx (or Field Water Production Total) over the specified period of production for both models. The difference in both the numerical and analytical method is due to truncation error or approximation scheme used in the eclipse software. But regardless, both methods give good predictions.

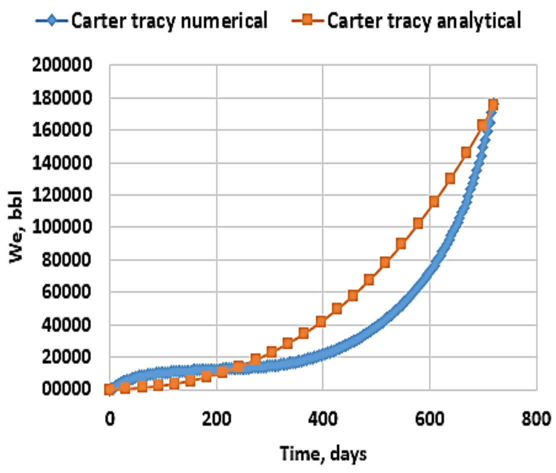

Fig. 9 Cumulative Water Influx for Carter Tracy Model (Numerical) Vs. Carter Tracy Model (Analytical)

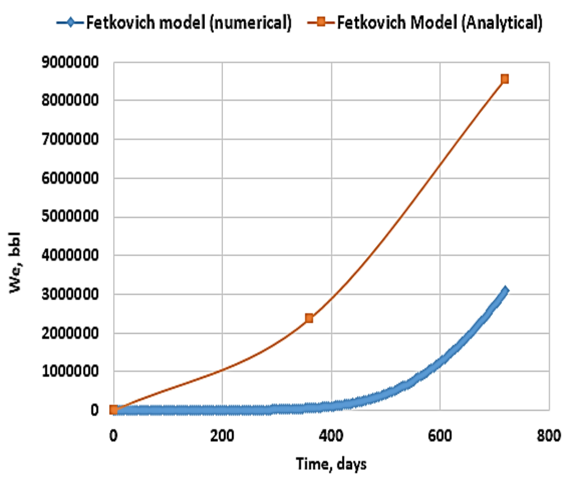

Fig. 10 Cumulative Water Influx for Fetkovich Model (Numerical) vs. Fetkovich Model (Analytical) [720 days]

As observed from Fig. 10, there is a huge difference between the cumulative water influx for Fetkovich model (analytical) and Fetkovich model (numerical) at the end of 720 days. The water influx recorded at the end of 720 days for the Analytical Fetkovich model was 8 552 092 bbls while that of its numerical equivalent model was 3 086 579 bbls. This indicates a variance of about 5 465 450 bbls of water. In contrast to the results obtained at 720 days, when both methods of the Fetkovich model were allowed to run for a longer duration (1460 days), the cumulative water influx for both methods appears to be more identical as shown in Fig. 11. Both the analytical and numerical models of the Fetkovich aquifer model attained a peak water influx value of approximately 26 635 592 bbls at the end of 1460 days simulation period. This ultimately implies that, longer production periods from reservoirs bounded by Fetkovich aquifers yields similar water influx results.

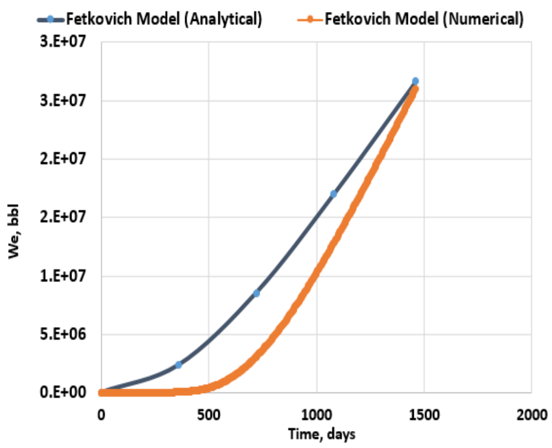

Fig. 11 Cumulative Water Influx for Fetkovich Model (Numerical) vs. Fetkovich Model (Analytical) [1460 days]

3.4 Carter Tracy Versus Fetkovich Numerical Aquifer Models

In order to ascertain the degree of uncertainty that exists between the two main numerical models (Carter Tracy and Fetkovich) in this research, the cumulative oil production and cumulative water influx curves of the two models were superimposed and shown in Figs. 12 and 13, respectively.

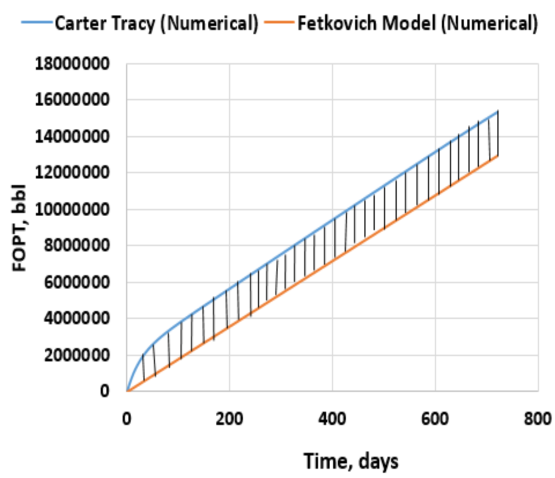

Fig. 12 Cumulative Oil Production for Fetkovich Model (Numerical) vs. Carter Tracy Model (Numerical) [720 days]

It is observed from Fig.12 that, there is fair degree of error (indicated by the shaded portion) existing between the Carter Tracy and Fetkovich numerical models, in terms of cumulative oil production. This degree of error between the two models is constant for oil production within a period of 720 days of simulation and could be attributed to the different assumptions made in each model formulation.

The results from the overlay of the Carter Tracy and Fetkovich numerical models in terms of cumulative water influx indicates an exponentially increasing Fetkovich water influx relative to Carter Tracy water influx model (Fig. 13). This is in line with the fact that, a large pressure differential is quickly developed between the undersaturated reservoir and the aquifer. This in turn promotes rapid water influx into the reservoir causing huge water influxes. From Fig. 13, it is seen that these two numerical models have an expanding error margin between time as the production days increase.

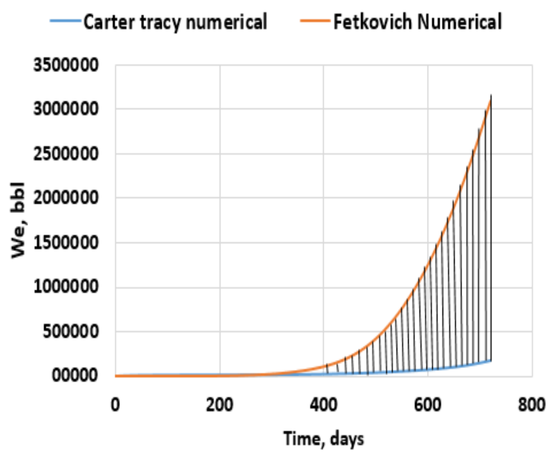

Fig. 13 Cumulative Water Influx for Carter Tracy Model (Numerical) vs. Fetkovich Model (Numerical) [720 days]

4 Conclusions

At the end of this study, it was observed that the numerical approach (eclipse) yielded more realistic and accurate results since it considered most of the reservoir heterogeneity parameters. The Van Everdingen model was not incorporated in the eclipse software, therefore Carter Tracy model, which appeared to be an approximation of the Van Everdingen model after the computation in MATLAB, was used for the comparison analysis since it could be modelled using the Eclipse 100 software. In addition to the above, the numerical approach (Eclipse model) yielded a closer time step of results since it could be manipulated (fine tuning) in the Eclipse 100 software.

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